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THE INTUITIVE UNDERSTANDING OF PROBABILITY CONCEPTS IN GRADES THREE AND FOUR

by

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A THESIS

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ABSTRACT

The purpose of this study was to investigate whether students in grades three and four have an intuitive understanding of six concepts of probability. The concepts tested were: sample space-single outcome event, sample space-ordered pair outcome event, equally likely event, more likely vs. less likely event, impossible vs. certain event, and probability of a simple event. Variables studied as to their relationship to subjects' performances of the probability concepts were: age, grade, sex, verbal IQ, nonverbal IQ, mathematics concept achievement, and mathematics problem solving ability. Each concept was tested in each of three instructional modes: concrete, pictorial, and symbolic.

The sample used in this study consisted of 60 grades three and four students of a suburban elementary school. The sample was selected to ensure 30 girls and 30 boys. All 60 students who were selected were able to complete all aspects of the testing.

Standardized tests, The Lorge-Thorndike Intelligence Tests, Level 3, Form A, Verbal and Nonverbal Batteries and the mathematics section of the Canadian Tests of Basic Skills-Form 1, were administered to the students as a group. From the results of this testing the individual IQ scores and mathematics achievement percentiles were obtained. Subjects' intuitive understanding of the probability concepts was tested by means of a 36 question probability test. This test was administered by means of a personal interview with each subject. All discussion was audiotaped for later analysis.

One-way analysis of variance and statistical correlations were used in the analysis of the resulting data.

The analysis revealed that grades three and four students had an intuitive understanding of four of the six concepts. They were: sample space-single outcome event, equally likely event, more likely vs. less likely event and impossible vs. certain event. The variables of age, grade, and sex were not found to be significant in determining subject's performance on the probability test. Subjects' IQ and mathematics ability were found to be highly correlated to their overall probability test scores. The different mode sequences had no significant effect on subjects' total test scores.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter 1

INTRODUCTION TO THE STUDY

GENERAL INTRODUCTION

The notion that certain concepts of probability can be and should be included in elementary mathematics curricula has been supported by recent research studies. Squire (1978) states "that provision be made for informal consolidation of existing concepts and ideas (of probability) for grades one and two and a wider range of activities and experiences from grade three onwards in which students are led to further concepts and into quantification of probability." (abstract)

Cohen (1957) claims that the system of education used in today's schools does not give students the opportunity to develop skills in recognizing degrees of uncertainty but leaves them with the impression that every question has a single definite answer. He states, "Our everyday speech is made up in large part of words like 'probably', 'many', 'soon', 'great', 'little'. What do these words mean?" (p.132) If education today is to fulfill its goal, of preparing citizens to live in a society that is both uncertain and pluralistic by its very nature, it must be prepared to accept the challenge that certain probability concepts can be developed and taught in the elementary mathematics program. The question arises, do students in the lower and middle elementary grades have the intuitive background to grasp the concepts and methods of probability theory? Research says YES.

BACKGROUND OF THE PROBLEM

> In the education of children the implanting of important mathematical ideas should take place as early as possible. This principle holds especially for the study of chance phenomena. Probability theory is one of the most multifaceted branches of mathematics, and its study requires an introduction to a large number of new and unusual concepts with which pupils need time to become familiar. (Engel, 1966, p. 771)

Piaget and Inhelder (Translation, 1975) claim that students' development of notions about probability begin as early in life as four years. This early developmental stage continues to formulate until the age of seven. The second developmental stage occurs between the ages of seven and eleven. Most students will have completed grade five by the end of it. Their (Piaget and Inhelder) experiments indicated that during this stage students do understand probability concepts. They concluded that the ideas of random mixture, regular and irregular distributions are developed during this stage.

Ross (1966) and Hoemann and Ross (1971) concur with Piaget and Inhelder's conclusions. They (Hoemann and Ross) found that students by the completion of grade four have mastered probability concepts which consisted of single-array tasks and comparable two-array tasks.

Offenbach (1964, 1965) determined that the sequential nature of probability tasks was understood by grade four students. He also concluded that students in grade four tried to find a "rule" that would govern the occurrence of events. This allows students to determine simple probabilities (often referred to in literature as the quantification of probability).

Several survey studies have been conducted by graduate students which investigated whether elementary students have intuitive under-

standing of specified probability concepts. Doherty (1965) determined that students in grades four, five, and six have acquired considerable intuitive understanding in four probability concepts tested. Leffin (1968) studied three concepts with students of grades four through seven. He found the students were able to understand and apply the concepts tested. Mullenex (1968) also concluded that selected concepts of probability should be included in the elementary mathematics curricula.

Jones (1974) tested five probability concepts and Squire (1978) tested six concepts with students from grades one, two, and three in independent studies. Both, however, concluded that an intuitive understanding of probability concepts is present at these grade levels and instruction in probability could be introduced into the curriculum beginning at grade one.

Shepler (1969), Gipson (1971), and McLeod (1971) have successfully taught units of probability mathematics to elementary students. Their studies involved students from grades two, three, four, and six. Each concluded that successful probability units could be developed and taught at the respective grades.

Emanating from the research, reports have been developed which indicate that the inclusion of probability into the elementary curriculum is feasible. The Cambridge Conference on School Mathematics (1963) concluded that probability should become a vital and appropriate component of the elementary mathematics curricula. Widespread acceptance for such an inclusion was reported by the National Advisory Committee on Mathematical Education (1975).

There is, however, a noticeable absence of these notions from

approved elementary mathematics curricula in most of North America. This apparent lack of interest is not the result of a lack of prepared materials, however. Two examples: the Macmillan Company's published textbook, What are the Chances (1963) and the School Mathematics Study Group publications for teachers and students, Probability for Primary Grades (1965, 1966), provide an indication of what is available. The fact that probability does not appear in the majority of elementary mathematics programs seems to be a direct result of the decision making policies of curriculum writers. Therefore the majority of elementary students still do not have the opportunity to study probability. Cohen (1957) claims that this results in students not being able -" in short, to interpret and become masters of their own uncertainties" (p. 138).

STATEMENT OF THE PROBLEM

The purposes of the present study were: (1) to determine if grade three and four students have an intuitive understanding of six basic probability concepts, (2) to investigate the effect of the factors, age, grade, and sex on the subjects' performance of the probability tasks, (3) to examine the effect of the mode embodiment of the probability test questions on the subjects' performance of the probability tasks, and (4) to determine if the factors of IQ and mathematical achievement can be used to predict subjects' performance of the probability tasks.

The first purpose was to consider the following six probability concepts:

A. sample space - single outcome,

B. sample space - ordered pair outcome,

C. equally likely,

D. more likely vs. less likely,

E. impossible vs. certain,

F. probability of simple events,

Definitions of these concepts are contained later in this chapter.

The second purpose was to consider the effect that age, grade, and sex may have on the learning of probability concepts. Their effect on student performance was investigated as part of this study.

Bruner (1964) proposed a three stage model of intellectual development: enative, iconic, and symbolic. These stages are referred to as concrete, pictorial, and symbolic in this study. Bruner (1967) further generalized that an optimal sequence of instruction would employ all three stages. Kieren (1978) suggested the need for more research on the use of manipulative materials on the effectiveness in teaching mathematics at all levels of education. Higgins (1976) felt that educators had a general agreement on the use of physical objects in mathematics education, particularly for younger children. A third purpose of the study was to examine the effect of these stages or mode embodiments.

The fourth purpose was to determine if students' verbal and non-verbal IQ scores and mathematics achievement percentiles could be used as valid predictors of their performance on probability tasks. No attempt was made to determine the effect of such factors as socio-economic status and home environment. Neither were factors such as vocabulary, reading ability, listening ability, and visual ability

considered in this study. Many of these factors would be worth examining in further research projects.

DEFINITIONS OF TERMS

For the purposes of this study, the following terms will be used as defined.

Verbal IQ - The measure of an individual's ability to understand ideas expressed in words.

Nonverbal IQ - The measure of an individual's ability to understand ideas expressed in non-word symbols.

Mathematics Concepts - The notions or ideas in mathematics that relate to the understanding of the number system, mathematics terms, and operations.

Mathematics Problem Solving - The notions or ideas in mathematics that relate to the finding of mathematical solutions to written mathematical problems.

Probability Concepts - The notions or ideas in mathematics that relate to determined outcomes of chance or of random selection.

Sample Space - Single Outcome Event (Concept A) - The identifying of any one of several possible solutions of a probability question.

Sample Space - Ordered Pair Outcome Event (Concept B) - The identifying of any one of several possible ordered pairs which constitute a solution of a probability question.

Equally Likely Event (Concept C) - The identifying of alternative outcomes which could occur with equal frequency.

More Likely vs. Less Likely Event (Concept D) - The identifying of specified alternative outcomes which are more likely or less likely to occur.

Impossible vs. Certain Event (Concept E) - The identifying of a specified alternative outcome which could never occur (impossible event) or which would always occur (certain event).

Probability of a Simple Event (Concept F) - The identifying of the probable frequency with which a specified alternative outcome would occur.

Mode Embodiment - The concrete, pictorial, or symbolic representations of the probability questions.

Concrete Mode (Mode I) - The use of manipulative materials such as a spinner, some marbles, several blocks, etc.

Pictorial Mode (Mode II) - The use of pictures of the manipulative materials.

Symbolic Mode (Mode III) - The use of verbal descriptions of manipulative materials.

HYPOTHESES TO BE TESTED

Null Hypotheses

1. Given subsets of scores of the probability test, as determined by age, grade, and sex, there is no significant difference between the means of those scores.

2. There is no significant relationship between the subjects' scores on the probability test and their respective scores on standardized tests for verbal IQ, nonverbal IQ, mathematics concepts achieve-

ment percentile.

3. Given subsets of scores of the probability test, there is no significant relationship between the subjects' obtained scores in the concrete mode, the pictorial mode, and the symbolic mode when considered in pairs.

4. Given subsets of scores of the probability test, there is no significant difference between the overall mode sequence mean and the mean in each of the six mode sequences.

5. Given subsets of scores of the probability test, there is no significant relationship between the subjects' obtained scores for concepts A through F when concept scores are considered in pairs.

LIMITATIONS OF THE STUDY

1. Personal factors: There was no attempt to determine the influence of socio-economic status, motivational level, or the effect that different social experiences may have on the responses of rural and urban students.

2. Motivational factors: There was no deliberate attempt to motivate the subjects. At the beginning of each interview a short explanation was given to each subject that the discussion was to be audiotaped. This may have had some motivational effect.

3. Instrument factor: The probability test could be regarded as a limitation of the study since it was designed by the researcher and it was not validated.

4. Sampling factor: The sample for the study was selected from one school in a large metropolitan area. The subjects were

selected by the administration of the school on the basis of equal numbers of male and female. No further attempt was made to randomize the sample for a large population.

ORGANIZATION OF THE REPORT

Chapter 1 contains the overall and specific statement of the problem to be studied. A review of the literature pertinent to the study is contained in Chapter 2. Chapter 3 contains a detailed description of the experimental design, the IQ tests, the mathematics achievement tests, the development and administration of the probability test, research procedures and the statistical design used to test the hypotheses. Chapter 4 presents the data that resulted from the study and all analyses of the data. The fifth and final chapter includes a summary of the study, a discussion of the conclusions, implications and suggestions for further research.

Chapter 2

REVIEW OF RELATED LITERATURE

Is probability too abstract for use in the elementary school? Do children believe that there is an exact answer for all mathematical questions? How often are children asked to interpret terms such as: maybe, sometimes, likely, perhaps, etc. These questions and others have led researchers to examine the value of probability for the elementary school student. Should not the educational system equip the child with the basic skills and ideas of probability?

Research indicates that students of ages four and upward can show interest, enthusiasm, and a degree of understanding toward basic concepts of probability given the appropriate situation. Bruner (1962) quoted Inhelder (Piaget's associate) in support of the readiness concept in early childhood.

> The teaching of probabilistic reasoning, so very common and important a feature of modern science, is hardly developed in our educational system before college. The omission is probably due to the fact that school syllabi in nearly all countries follow scientific progress with a near-disastrous time lag. But it may also be due to the widespread belief that the understanding of random phenomena depends on the learner's grasp of the meaning of the rarity or commoness of events. And admitted such ideas are hard to get across to the young. Our research indicated that the understanding of random phenomena requires, rather, the use of certain concrete logical operations well within the grasp of the young child -- provided these operations are free of awkward mathematical expression.

Thus it appears that students in the elementary grades do have the needed readiness factor to study probability.

Concept Development Stages

For purposes of clarity, the research reviewed here will be divided into three sections. Each of these three sections will deal with research connected with an age group of subjects as determined by the research of Piaget and Inhelder (Translation, 1975). Piaget and Inhelder divided students' development of understanding of probability notions into three stages:

(a) The first stage - four to seven years

(b) The second stage - seven to eleven years

(c) The third stage - eleven to twelve years

A. First Stage

Piaget devised three experiments to test subjects aged four to seven years.

The first experiment involved an open rectangular box which rested on a transverse pivot allowing it to seesaw. Eight red balls and eight white balls would come to rest at the declined or resting end. Each subject was told and shown how the box tipped, although the balls were held in place. He was then asked to predict the position of the balls if the box were tipped. This procedure was repeated a large number of times. All subject responses were recorded.

Piaget's second experiment used a heads and tails idea. The discs consisted of white counters with a circle on one side and a

cross on the other side. The number of counters thrown at one time varied in number from 10 to 20 at the experimenter's discretion. The results of each throw were recorded in number of circles and number of crosses. After each throw the subject was to predict the next throw's outcome by using the results of previous throws. Then without letting the child realize the substitution, the experimenter threw about 15 counters having the cross on both sides. This served as a control factor in the experiment. The experiment was repeated until a sufficient understanding of the child's reaction was obtained.

A sack of twenty red and twenty blue marbles served as apparatus for the third experiment. The child was asked several questions on how the marbles would appear if drawn randomly. Then he was given a sack with only blue ones. This matched the situation in the second experiment in that the counters in the second set were similar in appearance and number to those in the first set except they had crosses on both sides. In the sack of blue marbles, there was simply an absence of the red marbles which the child expected to be there. This type of control often gives the surest index of the judgment of probabilities of which the child is capable (Piaget, trans. 1975, p. 97).

In analyzing responses given by the subjects in the first experiment, Piaget concluded that they either denied the progressive mixing of the balls or thought of it as too regular. Either response avoids the idea of change. He also stated that "the child does not suspect the true nature of random mixture and tries constantly to find within the disorder, which he considers as only apparent, a

hidden order of some kind based on the common properties of the elements" (Piaget, trans. 1975, p. 218 - 219). As a result of experiments two and three, Piaget concluded that children at this age will give predictions that appear to be based on the probability concept of compensation. However it is an intuitive notion based on a theory of "each one gets his turn". When confronted with trying to explain why the counters turned up all crosses (experiment two) and similarly why all the marbles in the second sack were blue (experiment three), the students referred to the new situation as a trick or showed signs of confusion. Piaget explained this reaction as the child's belief in a hidden order which is made manifest in the frequency of event A or event B. Stating this another way, the child will most likely make his choice on the basis of the initial order of marbles (I'll get red because it's first), ignoring the mixing of elements in the bag (Yost, Siegel, Andrews, 1962).

Recent research has had a tendency to negate the work of Piaget, provided the child had an appropriate environment within which to work. Some studies revealed a positive factor when a form of reward was incorporated.

In a comparative study with deaf and hearing children as subjects, Ross (1966) stated that children of this age do have some probability knowledge in favor of prevailing majority odds. In his study he approached the problem as a puzzle. A large heavy black cardboard box was placed before each subject. The examiner then placed into the box a predetermined number of yellow balls and green balls. The subject was then asked to pick a ball from the box by reaching through

a flap. Before he actually picked the ball, he was asked to predict the color of the ball picked. He could do this by pointing to the yellow ball or the green ball previously fastened to a board in front of him or he could vocalize the answer. A casual examination of the predictions of the seven-year-olds (16 subjects), showed that they simply altered color prediction. Ross (1966) concluded, that "present results make clear that children who alternate predictions are not consistently making a connection between the previous event outcome and the following prediction but instead alternate predictions without regard to previous event outcomes." It can also be noted that the child who alternates his predictions does so because he believes the different colors take turns. This is in conflict with Piaget's belief that children predict on initial order. Ross further claimed each prediction was considered as an independent event. Even children, chronological age 7, showed some understanding of prevailing majority odds.

In a later study Hoemann and Ross (1971) studied children with mean ages (year - months) 4-7, 6-0, 7-5, 10-6, in an experiment which was designed to use discs and spinners. The discs were constructed of paper, 12 inches in diameter, having different patterns of black and white wedges. Each subject was given 2 discs and he was required to choose a disc he preferred to spin for the color named by the examiner (probability task). The discs were changed with every question. The 20 pairs of discs were stacked on 2 pieces of plywood in front of the subject. Of the 36 questions, half the questions had the left disc as the correct choice and half the trials had the right

TABLE 1

MEAN PERCENTAGES CORRECT BY CHRONOLOGICAL AGE

(HOEMANN AND ROSS, 1971)

Odd differences	Chronological age			
	4 - 6	6 - 0	7 - 6	10 - 6
1/2	83	93	98	100
1/4	80	87	94	97
1/8	62	82	89	93
combined	75	87	94	97

disc. In addition half the discs were solid black or solid white while the remaining discs had alternating black and white segments.

TABLE 2

MEAN PERCENTAGES CORRECT BY CONTROL FACTOR

(HOEMANN AND ROSS, 1971)

Odd differences	Chronological age			
	4 - 6	6 - 0	7 - 6	10 - 6
1/2	92	99	100	100
1/4	80	95	97	99
1/8	72	88	94	96
combined	81	94	97	99

To serve as a control to the experiment, Hoemann and Ross asked each subject questions based on proportional tasks. The subject was to indicate which of the 2 discs had the most black (white) in it. For these questions the spinners had been removed from the discs.

The data show that the understanding of concepts of probability does not necessarily contribute to correct solutions since they can be predicted from the non-probability task.

Therefore Hoemann and Ross designed a second experiment using twenty subjects, 10 boys and 10 girls in each of seven age groups. Each age group had limits within six months of the mean ages 4-3, 5-3, 6-2, 7-2, 8-2, 12-2, and 13-2, (140 subjects). The task was for the subject to indicate nonverbally with a single spinner which color the pointer would point to when it stopped, black or white. Each subject was also instructed in a proportional task to indicate which color, black or white, comprised the larger total area. As in experiment I, the odds differences were, 1/2, 1/4, 1/8, the colored wedges were varied, and a third sequence was added in which a random ordering of the three fractional variables, odds, differences, number of wedges, color of correct response, was selected. The mean percentages correct for this experiment are given in table 3.

Two specific results should be noted. The first is that there were significantly more correct responses in the combined proportionality results as compared to the combined probability results at each age from 4 through 8. Secondly this suggests that the single spinner task does test for probability concepts. From this interpretation it is valid to claim that the subjects with CA 4 were

TABLE 3

MEAN PERCENTAGES CORRECT (HOEMANN AND ROSS, 1971)

Odd differences and task		Chronological age 4	5	6	7	8	12	13
1/2	Proport.	74	97	100	99	100	...*	...*
	Prob.	52	69	81	85	82	94	96
1/4	Proport.	70	92	98	97	100	...*	...*
	Prob.	56	66	78	81	76	86	87
1/8	Proport.	68	82	99	93	98	...*	...*
	Prob.	61	57	71	70	73	84	88
Combined	Proport.	70	90	99	97	99	...*	...*
	Prob.	56	64	77	79	77	88	90

* Scores assumed to be 100 but not tested.

applying concepts of probability. This also casts doubt on Piaget's contention that young children cannot understand probability.

In two successive studies (1964, 1965), Offenbach also indicated areas of doubt in regard to Piaget's beliefs on the lack of ability of small children to understand the factors of probability. His first study was performed with 30 kindergarten children (first stage) and 30 fourth-grade children (second stage). A 3 X 2 factorial design (three reward-punishment groups and two age levels) permitted random placement of 10 subjects in a group. A 0 - 0 group received no reward or punishment other than the informational feedback of confirmation or disconfirmation of their predictions. A 1 - 1 group received one marble for correct guesses and lost one marble for incorrect guesses;

children in a 3 - 3 group received three marbles for correct guesses and lost three marbles for incorrect guesses. The main observation of significance is that the kindergarten children's predictions were based on the immediately preceding event. These children appeared to be unaware of the possible sequential nature of the task. This behavior was consistent with Inhelder's and Piaget's description of behavior in the first stage of logical thinking. Offenbach felt that this result was due to the method of stimulus presented and this led him to do his second study.

He now chose a larger sample of 72 kindergarten children (first stage) and 72 fourth-grade children (second stage). The experimental design (a 3 X 2 X 2 factorial) permitted random placement of 12 subjects in each group. A further division of tasks into successive tasks and simultaneous tasks was set up.

In comparing the results of this experiment with developmental theories of Piaget and Inhelder, Offenbach draws several conclusions.

1. Kindergarten children probably have a rudimentary understanding of the concept of probability.

2. Piaget's theories (concerning probability concepts only) cannot be rejected by examining probability matching behavior alone.

3. Piaget's methodology appears to be dependent more on the recognition of probability factors than on the ability to discriminate frequencies. This recognition is a prerequisite to the understanding of probability concepts.

This last conclusion is also supported by the data provided by

the work of Ross, previously referred to.

A group of researchers (Andrews, Seigel, and Yost, 1962) after studying the work of Piaget and Inhelder on probability in children developed a decision-making method for assessing understanding of probability. The criticisms of Piaget's work that Yost et al. tried to overcome by their design are:

1. Piaget's method relied heavily on verbal skills.

2. Piaget's method confounds color preference with color expectation.

3. Piaget placed before the subject an assortment identical to that in the bag, in the interests of aiding his memory, but this in fact may have been confusing because the assortment was not randomized.

4. Piaget's method provided no special incentive for a correct response.

5. Piaget made no provision for statistical analysis of the results.

The design that was selected differed from that of Piaget's in the following ways.

1. There was little reliance on verbal skills. The subject simply had to point to the box he chose.

2. The influence of color preference was controlled.

3. By utilizing transparent boxes, the need for a duplicate set of chips was avoided.

4. High motivation for correct responses was assured by the use of prizes.

5. A large number of trials and repeated trials were used in

order to apply statistical techniques in determining whether or not the subject did show evidence of an understanding of probability.

The twenty subjects selected, 10 boys and 10 girls, ranged in ages from 4-10 to 5-8. Each subject participated in two sessions on two separate days.

The child's score was determined by the number of correct responses he made in a series of 24 trials. A score of 12 represented a performance at only a chance level, while a score of 24 represented a performance consistent with the concept of probability. The median number of correct responses over two sessions on the decision making condition was 18.

Yost et al. concluded from the analysis of the data, that below the age of 7 and even as low as 4 years, subjects have some understanding of probability under situations where controls are introduced, reinforcement is increased, and a nonverbal response is adequate. The major limiting factor in the research of Yost et al. is that the number of trials in each session was too few to permit meaningful within-sessions analysis of change.

A replication of the Yost et al. (1962) experiment was conducted by Goldberg (1966) with minor modifications. These modifications included differences in sampling of distributions and details of procedure. It is therefore not necessary to deal in detail with this study. It is however of interest to consider Goldberg's conclusions. She stated that 4-5 year-olds do involve probability judgments although these judgments may be strongly influenced by color preference, by confusion of number, and by proportion.

Recent research has provided evidence which questions the conclusions Piaget formulated as to the ability of children under the age of 7 to understand concepts of probability. It has shown that under appropriate conditions children of this age can use probability concepts to varying degrees of competence.

B. Second Stage

Based on the experiments discussed in section A, Piaget concluded that children from age 7-11 years do understand probability concepts. Experiment one resulted in Piaget's concluding that the main characteristic of these subjects (second stage) is to believe in a real and progressive mixture. He further divided stage II into substages A and B by separating the idea of randomness into a simple realization of existence and a refinement of the idea (Piaget, p. 18, translation 1975).

In the experiment using the counters "the most striking characteristic of the reactions of stage II is the refusal to accept the results of the experiment with the false counters" (all crosses) (Piaget, p. 18, translation 1975). Because of this previous experience, stage II subjects understood that the sack was filled solely with blue marbles. It was easy for them to grasp intuitively the absence of red marbles. The older the subjects, the more quickly the understanding developed.

Piaget's general conclusions of this stage (Piaget, p. 229-230, translated 1975) of concern are that:

1. the idea of random mixture was globally acquired at a limited level (using small numbers),

2. a beginning of inductive reasoning was indicated. Its function consisted of distinguishing between regularities and fortuitous distributions,

3. when an extremely unlikely distribution occurred, an inductive search for a cause to explain such distributions other than by chance was indicated,

4. probabilistic comparisons between two varying groups was impossible.

Recent researchers have studied students at this stage, with comparative results. Ross gave considerable support to the conclusions put forth by Piaget (Ross, 1966; Hoemann and Ross, 1971). He concluded that at stage II subjects cannot effectively consider successive outcomes when linked together. Ross supported Piaget's conclusion that it is easier to choose between two arrays than to predict with a single array. In addition, Hoemann and Ross, with a greater concentration on this comparison have extended Piaget's conclusion. Piaget considered the one - array versus two - array comparison only for a few children with an age range from 6-1 to 7-6. Ross extended this range in both directions.

He further concluded that children with CA 7-6 to 11-0 attained nearly 90% success in experiment 1 with the single-array task and the comparable modified two-array tasks, but in experiment 2, 90% success was not reached until the onset of the formal operational period at CA 11-0 to 13-0 (stage III of Piaget). In contrast to the data of

Piaget and Inhelder that indicates the "discovery of chance does not occur until the onset of stage II", the data Ross collected indicate that much of the child's first envisioning of a concept of chance is almost completely mastered by this age. Ross goes on to say that this difference may in fact be due to a matter of definition as to what one calls probability knowledge.

In the two studies of Offenback (1964, 1965) he used, along with the subjects drawn from kindergarten, an equal sample from grade four. The results of this grade four sampling can be used to compare his conclusions with Piaget and others at the second stage level. Offenback concluded that age was not a significant variable in the concept of probability matching performance but that the older children (grade four) did try to find a "rule" governing the occurrence of the two events, indicating that they were sensitive to the fact that the stimuli might be interrelated in some way. His data also indicated that the behavior of the older children was more appropriate from a probability viewpoint and that they were more sensitive to the sequential nature of probability tasks. These conclusions are also supportive of the findings of Piaget, which were previously discussed in this stage.

The studies of Andrews (1962), Goldberg (1966), Seigel, and Yost did not use subjects beyond the stage I level.

In summary of the related studies of Piaget and Inhelder, Ross, Hoemann and Ross, and Offenback, a closer similarity can be noted between the conclusions drawn for stage II than was the case for stage I. It is therefore probable because of the more comparable

results in recent research that Piaget's identification of the characteristics found in children during the age range for stage II is relatively accurate.

C. Third Stage

The characteristics of children in the third stage (ages 11-12), as Piaget concludes, are as follows. The child's formal understanding of probability concepts appears about age eleven or twelve when he advances into the formal operational stage. It is at this time that the process of random mixture is understood. Piaget concludes that this understanding is due to the assimilation of the observed facts in an "operative scheme based on the mechanics of permutations" (Piaget, p. 23, translated 1975). Piaget also found that students of this stage have "finer judgments". This understanding goes beyond the global effect in stage II and the concept is now broken into gradual judgments. Also shown here are increasing relativity and the subjects' recognition of the sense that there are many different forces at play as opposed to a simple attributive judgment.

Other characteristics of the third stage subjects indicated the ability to structure totalities which led to the law of large numbers. They also indicated the recognition that the probabilities of isolated cases were functions of the whole. Piaget further concludes only here (third stage) that the mechanism of reasoning was visible.

Other studies certainly support Piaget in his conclusions of this age. Ross (1966) and Hoemann and Ross (1971) in their studies, continued their survey through the operational stage (age approximates

in the first study of 11, 13, 15, and age means in the second study of 12-1, 13-2). Ross supported the findings of Piaget, that subjects must reach this developmental period before they can effectively consider successive outcomes as being linked together, or part of a whole. He further stated that subjects do not consistently relate predictions to consecutive event outcomes until CA 15, whereas Piaget felt that the mechanism of reasoning such outcomes was present at CA 11 and 12.

Using subjects aged 10-16 Cohen (1957) determined four stages for development notions of probability. He randomly drew four beads, 16 times, from a bowl filled with equal numbers of blue and yellow beads and placed them into 16 cups. The subjects were allowed to watch this procedure. They were then asked to tell how many of the cups would contain respectively: (1) four blue beads, (2) three blue and one yellow, (3) two blue and two yellow, (4) one blue and three yellow, (5) four yellow beads. He then stated that on the "basis of such experiments" we have found that the subjects' understanding of probability concepts progress through four stages. These four stages can be summarized into (1) subjects merely guess vaguely, (2) subjects realize that the most frequent content of the cups will be two blue and two yellow beads, (3) subjects realize that one blue and three yellow occur equally as often as three blue and one yellow, and that four blue beads and four yellow beads also have equal probabilities, (4) the subjects can conclude that a one to three combination will occur more frequently than a four bead one color combination.

His study went on to discuss other related experiments. It is

wise to caution against the direct application of this study as there are no recorded data or methods on how the conclusions were derived. Mention of the study was simply to serve as a general confirmation of the understanding that subjects, CA 10 and higher, can perform with insight, controlled events of probability.

Classroom Studies

Three studies will now be discussed which are not directly involved with Piaget's three stages of development but which are concerned with the workability of probability concepts in the elementary school. The first of these studies was jointly reported by Wilkinson and Nelson (1966). Concluding that the experiences and ideas should be meaningful to the students, they divided the concepts of probability into three types of situations. First would be "personalistic" situations which were drawn from data involving the students' own environment. These data included such items as telephone numbers, birthdays, heights, shoe sizes, etc. A sample question was "Which student is about the middle of the class in height?" Second type situations (familiar situations) involved models in the elementary study of probability concepts found in the use of coins, die, and cards. Questions asked included "Is it fair to flip a coin to decide whether Jim or Joe gets top bunk?" The last situation type dealt with unfamiliar situations. Here items such as a paper cup, thumb tack, drawing samples of beads from a container, etc. were used. In relation to the paper cup, the examiner asked, "How many outcomes in flipping a paper cup? Which is most likely? Which is least likely?"

This study was conducted with a sixth-grade class, by the authors themselves for a three week period. There was no textbook used and no formal testing. The conclusions drawn were based on the subjective observations of the experimenters. Two of their conclusions were (1) "We feel that the most productive situations will be those that are unfamiliar to students," and (2) "It does seem important that during the elementary school years, children should be exposed to events which have a degree of uncertainty".

Shepler (1970) taught a class of 25 sixth-grade students a four-week unit in probability. It was his purpose to test the feasibility of teaching probability statistics to a class of sixth-grade students and to construct a set of instructional materials and procedures for such a unit. He designed fourteen behavioral objectives for the unit of study. A pretest and a posttest were administered. The pretest showed that no behavioral objective had a mean percentage above 80%, two objectives had means between 50% and 80%, and 12 objectives had average means below 50%. The results of the posttest showed that eleven objectives had averages above 90% and only one objective had an average below 50%.

The results of Shepler's first study must be considered in view of two conditions not normally found in the average classroom. The subjects selected had a mean IQ of 117.7 and a standard deviation of 6.8. Each classroom had two teachers at all times. These conditions prevent the use of generalizations from the results.

His second study (1973) followed a similar format with the addition of a retention test. The performance of the children on

the retention test, administered four weeks after the posttest, by objectives, was similar to their overall posttest performance. He concluded that the low initial performances on these two studies correlate with the findings of Piaget in that most of the students at this age have not yet acquired the formal-operational stage of development.

Dissertation Studies

This segment of research naturally divides itself into two sections. The first section will deal with studies that examine the status of certain basic concepts of probability possesed by children in the elementary grades. Section two will deal with studies that involved the teaching of probability concepts to students in grades K-6.

A. Probability Surveys

Research has indicated that children do acquire notions of probability before they receive any formal education. Researchers have been prompted by this to determine what basic probability notions are inherent in children and if the variables of age, sex difference, general ability, arithmetic ability, IQ, etc. are in fact significant.

Doherty (1965) tested 54 grade four, five, and six subjects to determine their intuitive understanding of four selected concepts of probability. The concepts selected were (1) the idea of a sample space, (2) the probability of a simple event in a sample space, (3)

the probability of the union of non-overlapping events in the sample spaces, and (4) the idea of the difference between mutually independent events and mutually exclusive events. She concluded that the subjects had acquired considerable understanding in dealing with the selected concepts. This understanding, she said, must have come from everyday experiences and not from formal education. The four concepts did not significantly differ in level of difficulty. Chronological age and sex did not significantly affect the subjects' familiarity with the concepts. Familiarity increased significantly with grade, teacher rating, mental age, arithmetic achievement, and general achievement.

Leffin (1968) sampled 528 grade four, five, six, and seven students to determine the status of three basic concepts of probability; (1) finite sample space, (2) probability of a simple event, and (3) quantification of probabilities. The subjects were categorized according to sex, IQ, and grade. A test of 10 or 12 items was constructed for each concept. Leffin found a significant relationship between the number of correct responses and IQ, and between the number of correct responses and grade level where high performance paralleled high IQ or grade seven and low performance paralleled low IQ or grade four. Sex differences only indicated a marginal significance on tests I and III. Since the subjects had no formal education in probability, the most significant outcome is the students' ability to understand and apply the concepts tested. He concluded this to be a result of their background, experience, and intuition.

Mullenex (1968) tested 109 grade three, four, five, and six students to determine if age, sex difference, general ability, and basic skill in school subjects could be used as predictors for the subjects' understanding of probability concepts. He constructed a probability test based on Piaget's work on chance concepts. The variables of age, general ability, vocabulary skill, work habits, and age were not found to be significant predictors. The F ratios indicated that there was little or no significant differences among the variables of sex, reading skill, arithmetic skill, and problem solving skill.

Although Doherty more clearly defines her recommendations, Mullenex and Leffin also agree that selected concepts of probability should be included in the elementary school program. Doherty cautioned against a rapid movement in this direction and stated the need for teacher preparation in the subject area.

Two more recent studies, Jones (1974) and Squire (1978) studied students in grades one, two, and three in suburban schools.

Jones took five concepts: (1) the outcomes of sample space, (2) most favorable event in a sample space, (3) most favorable sample space for a given event (event probabilities presented in the forms: a/n, b/n, c/n), (4) sample space equally favorable to a given set of events, and (5) the most favorable sample space for a given event (probabilities present in the forms: a/x, a/y, a/z). The 162 subjects were categorized by grade, IQ, and embodiment. The grade three subjects had better overall performance than that of either the grade one or the grade two classes. However their performance had a significant difference only to the performance of the grade one

class.

The performance of grade two subjects was also significantly higher than that of the grade one class. Within each grade, there were significant differences between the performances of IQ groups. High performance was paralleled with the high IQ group, and low performance with the low IQ group. Jones concluded that (1) children in the primary grades have begun to acquire some knowledge of probability and (2) that some concepts of probability could be introduced into the primary mathematics curriculum.

Squire's study determined the status of six concepts of probability, the level of quantification of probability present in the subjects, the performance difference due to the embodiment of the probability setting and the predictor effect of sex, grade, and IQ. The concepts selected were: (1) events in a sample space, (2) most favorable event, (3) most favorable sample space for a given event, (4) sample space equally favorable to a given set of events, (5) impossible event and (6) certain event. The performances of the 72 subjects indicated that concepts 1, 2, 3, and 6 were understood by at least 75%. The level of quantification of probability across the three grades had a mean of 42% correct. No significant effect was found due to the three embodiments of spinner, block, and box. Sex, grade, and IQ were all found to be of significance when considered on the total test. There was also found to be a marked increase in understanding of probability concepts as the subjects pass through grade three. Squire concluded that experiences and activities could be provided from grade three onwards which would develop further concepts of probability.

If the recommendations of Jones and Squire are added to the previous recommendations of Doherty, Leffin, and Mullenex, curriculum writers should give consideration to the inclusion of selected notions of probability throughout the elementary mathematics curriculum.

B. Classroom Studies

Several researchers have attempted to develop the concepts of basic probability in children. This was attempted through a sequence of instructional periods. Pretests, posttests, and retention tests were often used to test if the concepts were learned and retained. Gipson (1971) investigated the teaching of two concepts, finite sample space, and the probability of a simple event, to eight students from grades three and six. Two pilot studies were used. The first pilot study was used to determine the effectiveness of the materials to be used, the appropriateness of the concepts to be taught and the students' intuitive knowledge of probability. The second pilot study served to identify and evaluate a set of lessons that would be appropriate for elementary school children.

The individual-based instruction sequence for six students was audiotaped and the sequence for two students was videotaped. All tapes were later transcribed for detailed analysis.

Evidence collected from the pretests and posttests showed that both third and sixth grade students can learn these two concepts of probability. The author concluded that grade three was an appropriate grade to introduce selected probability concepts.

McLeod (1971) also used the experience gained in study A of

two parallel studies to modify instructional techniques for study B. An eight to ten day unit on probability was prepared and taught to second and fourth graders. Three experimental treatments, (LP) laboratory participation; (TD) teacher demonstration; and (M) no instruction between pretests and posttests, were assigned at random to classes of students at both grade levels in a single school. For study B, a control classroom (C) was added from another school. Although the character of the treatments was the same across both grades and common worksheets were used, the learning activities for grade two were less extensive.

The pretests indicated that most second graders as well as most fourth graders were able to use the concept of "likely" before receiving any instruction. Treatment groups LP and TD were significantly superior to group C at each grade level in study B on the early posttest measures. The treatment effect did not affect the learning level as indicated on the posttest measures or the retention measures. Reading ability, IQ, and sex did not have an effect on individual performances.

In an earlier study, Shepler (1969), gave support to the feasibiliby of teaching selected concepts of probability to elementary school students. For this study a unit was developed, based on the system of developmental modes, which encompassed the concepts of subjective probability notions, empirical probability, counting techniques, a priori probability including simple and compound events, and comparison of two events using probability. The unit was taught to a class of 25 sixth graders of average and above

average ability. To determine the feasibility of the unit, pre- and posttests were designed to measure the fourteen behavioral objectives of the unit.

The instructional treatment was concluded to be generally successful. The posttest percentage was 92.8% while the pretest was 37.9%. Eleven of the 14 behavioral objectives were highly mastered. The three unsuccessful objectives involved specifying the estimated probability, numbering the outcomes of an event, and estimating the probability of an event from the data. The author stated that the lack of mastery of these three objectives was a result of a lack of emphasis on the part of the instructor and insufficient practice given to the students.

Shepler, McLeod, and Gipson have successfully taught concepts of probability to elementary students ranging from grade two to grade six. Although Gipson used the personal interview approach to teaching, his study still gives support that students do have experiences which develop certain intuitive notions, based in probability. These notions can then be built upon by instructors into the more formal use of probability.

Probability in Curriculum

A number of publishing companies and study groups have printed probability units as supplements to curriculum or enrichment series since the early 1960's. A few references will be given below as a sampling of available programs. The Macmillan Company, (1963) developed a programmed instruction textbook, What Are The Chances.

It develops an understanding of what probability is and covers the topics: sample spaces and random choices, partitions and complements, conditional probability, independent events, and statistical probability. The authors do not indicate the grade level that it is designed for; however, because of the reading level it would probably fit best in the Junior High and upper elementary grades.

The School Mathematics Study Group (SMSG) has given considerable study to the matter (1965, 1966). Their work included Teacher's Commentaries and Student Texts. They covered two levels, namely Probability for Primary Grades and Probability for Intermediate Grades. This program covers a wide range of concepts too numerous for this reference.

Mathematics activities have highlighted more recent mathematics curriculum. Scott, Foresman (1971) published an activity oriented probability unit. However, no grade distinction or concept development was indicated.

Buckeye (1970) and Shulte/Choate (1977) have also attempted to get probability activities into the elementary schools by preparing activity oriented experiments for the students.

Curriculum writers and publishers have not shown a detailed outline of topics within probability that could be implemented at the elementary level of education. Harvey (1972), using task analysis, has attempted to provide a sample sequence chart for grades K-8. His selection of concepts involved 11 major concepts taken from statistics and probability. Behavioral objectives were then determined for each section and a task analysis chart was

constructed. It appears that the tasks involve more statistics than probability and are beyond the ability level of the student at each level as determined by the literature previously discussed.

Chapter 3

DESIGN OF THE STUDY

DESCRIPTION OF THE POPULATION

The population for this study consisted of grade three and four students of varying levels of achievement and socio-economic background. These students receive instruction in regular classrooms operated by public school systems, separate school systems, and parochial or private school systems. The curriculum being taught in these schools has been approved by the regional Department of Education.

DESCRIPTION OF THE SAMPLE

The sample was selected from the grade three and four students enrolled in Brookwood Elementary School, a suburban school just outside the confines of the City of Edmonton. The school's enrollment came from a rapidly developing community, the local farming community, and families living on small acreages. It was assumed that by obtaining a sample from this setting, a representative sampling of subjects from middle and low socio-economic status families was obtained.

SELECTION OF SAMPLE

The principal of the school was contacted and asked to make 60

students available to the researcher from grades three and four. Thirty male and 30 female students were requested.

There was no attempt made to ensure that equal numbers of students were selected from the two grades.

When the researcher arrived at the school for data collection, the list of pupils had already been prepared.

THE TESTING INSTRUMENTS

Three testing instruments were used in the collection of data:

1. the Lorge-Thorndike Intelligence Tests.
2. the Canadian Tests of Basic Skills.
3. the probability test.

The Lorge-Thorndike Intelligence Tests

These are a series of tests designed to determine the ability of an individual to work with ideas and the relationships among them.

The two specific test batteries used were:

1. The Lorge-Thorndike Intelligence Tests, Level 3, Form A, Verbal Battery, re-usable edition.

2. The Lorge-Thorndike Intelligence Tests, Level 3, Form A, Nonverbal Battery, re-usable edition.

The Verbal Battery was used because most of the ideas individuals deal with are expressed in verbal symbols. It was hoped that a relationship could be established between this type of expression and the overall score on the probability test. The Nonverbal Battery was used because of the presence of slow learners who may be restricted

in their verbalization of ideas. It was assumed that by including this battery, such students would not be penalized in any way.

Each of these two standardized tests required the use of distinct answer sheets. These were utilized in the study and students were given instruction on their use. These sheets were then hand scored using mask scoring keys which were obtained from the publisher (IBM H90916 and IBM H90917).

The Canadian Tests of Basic Skills - Form 1 (CTBS)

The CTBS are used to determine how well students have mastered the basic skills. The areas tested are: vocabulary development, reading comprehension, the mechanics of written expression, application of special reading techniques to work-study materials, and mathematical understanding. The basic skills associated with mathematics understanding are evaluated in two tests, mathematics concepts and mathematics problem solving. These two tests were the only ones administered from this battery.

Students were instructed how to use answer sheets to record their responses. These answer sheets were later scored using an answer key, provided by the publisher.

The Probability Test

The probability test was designed by the researcher. The design allowed for the testing of six concepts in each of three representations or modes. This resulted in a 6 X 3 matrix having 18 cells (Table 4).

Two questions were written for each cell. These questions were to be asked in sequence.

TABLE 4

CONCEPT MATRIX

Concepts	Modes I	II	III
A			
B			
C			
D			
E			
F			

Instructional Modes

The works of Piaget, Bruner, and numerous other research studies referred to in chapter two suggest that children acquire some notions of probability outside of a formal instructional setting. The hierachy of notions is presumed to progress through identifiable stages from: I, the concrete representation; II, the pictorial representation and; III, the symbolic representation. It has been further hypothesized that as the concept is developed progressively through the three modes, the learner shows a greater understanding of the concept.

The probability test was designed to test each of the six

concepts in each of three modes (see Table 4). Twelve questions were asked in each mode, two for each concept. A set of materials and apparatus was devised for the test.

Apparatus and Materials

Concrete Materials

Five sets of apparatus or embodiments were either purchased or constructed:

1. A selection of wooden beads was purchased at a local hobby shop. Each bead was approximately three centimetres long and two centimetres in diameter. The 20 beads consisted of five beads each of four colors; green, blue, brown, and yellow.

2. Eight wooden cubes (two cm X two cm X two cm) were painted in different ways.

    a. two cubes had each side painted one of six colors; blue, yellow, green, white, orange, and red.

    b. two cubes had opposite sides painted one of three colors with one cube having yellow, blue, and orange sides and the other cube having red, white, and green sides.

    c. two cubes had three adjacent sides painted one of two colors; green and white, and red and yellow respectively.

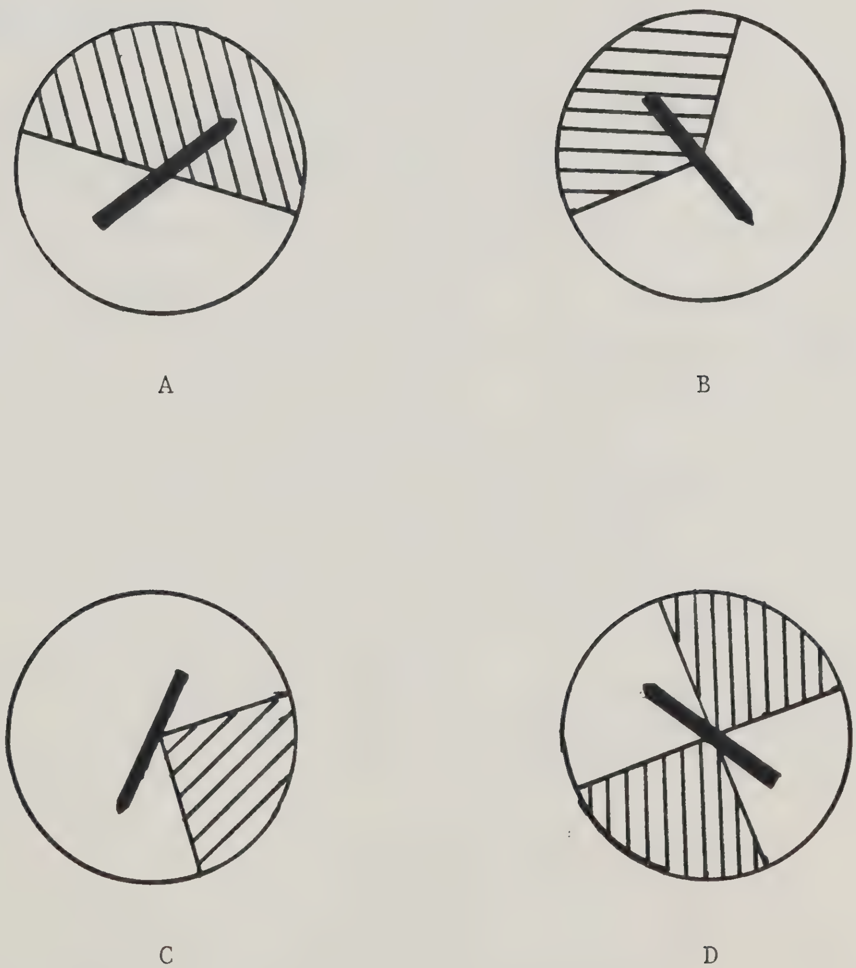
    d. two cubes had all six sides painted one color, either blue or orange.

3. Four plexiglass (plastic) spinners were constructed for the investigation. The round disc and spinner were mounted on

six mm sheets (23 cm X 23 cm) of white plastic. The four diagrams below indicate the color proportions of each spinner. The ratio for each spinner was: spinner A (one-half orange - one-half green); spinner B (one-third red - two-thirds yellow); spinner C (one-quarter yellow - three-quarters red); and spinner D (two-quarters orange - two-quarters green).

FIGURE 1

PROBABILITY SPINNERS



Each disc was approximately 20 cm in diameter. A red plastic arrow (1.5 cm wide and 15 cm long) was fastened to the center of each disc.

4. Two sets of six tennis balls each were purchased at a sports store. One set of balls was white and the other set of balls was yellow.

5. A selection of 40 large glass marbles (two cm in diameter) was purchased at a hobby shop. There were eight marbles each of five colors: clear, white, black, red, and yellow.

Pictorial Materials

One 8" X 10" (approximately 20 cm X 25 cm) color photograph of a specific set of apparatus was taken for each of the 12 questions asked in the pictorial mode. The apparatus was selected from the five sets of material described above.

The twelve pictures were numbered to correspond with the question number for easy retrieval (Appendix A). They pictured:

1. three red marbles and two black marbles,
2. spinner D, two-quarters - two-quarters,
3. three red marbles with one white marble, and a wooden cube with all shown sides painted blue,
4. spinner B, one-third - two-thirds and three white tennis balls with two yellow balls,
5. three brown beads and three green beads,
6. three red marbles and two black marbles,
7. five brown beads and two yellow beads,

8. spinner B, one-third - two-thirds,

9. six yellow tennis balls and six white tennis balls,

10. seven yellow marbles,

11. spinner C, one-quarter - three-quarters,

12. one wooden cube with one side yellow, one side orange, and one side blue being shown. The blue side appeared to be black on the photograph.

Each picture was placed in a clear plastic folder for protection of the picture surface.

Symbolic Materials

The 12 questions asked in the symbolic mode used only verbal symbolism. It was decided that no new embodiments would be introduced for these questions. The selection of a particular embodiment for each question will be discussed later under embodiment selection.

Concepts Selected

The research identified several basic concepts inherent to the study of probability. From them, six basic concepts were selected for use in this test.

A. Sample space - single outcome. The subject was expected to indicate that there may be several answers to a single probability task.

B. Sample space - ordered pair outcome. The difference between this concept and concept A is that there is a combination

answer. The subject was expected to respond in the order in which the outcomes occurred.

C. Equally likely event. Alternative outcomes could occur with equal frequency.

D. More likely vs. less likely event. A probability task was designed which favored one outcome over another outcome.

E. Impossible vs. certain event. The task was designed to ensure that the predetermined outcome could not occur (impossible event) or that the predetermined outcome would always occur (certain event).

F. Probability of simple events. The frequency of the identified outcome was to be expressed as a ratio.

The above concepts allowed for a wide range of probability tasks. The verbalization of the tasks could range in difficulty from everyday language; equal, impossible etc. to more probabilistic language; probability of an event etc. Each concept was questioned in each of the three modes. This allowed an environment to exist in which the developmental aspect of the modes could be examined.

Embodiment Selection

Several guiding principles were followed in the selection of the embodiments for the various tasks. They are that:

1. two embodiments would be used in each cell.

2. all five embodiments would be used for each concept and one embodiment would be duplicated.

3. twelve embodiments would be used for questioning concept

B to allow for the ordered pair outcomes.

4. the rotation of embodiments would also be used in the symbolic mode.

Table 5 indicates the selection of embodiments for each concept and mode.

TABLE 5

EMBODIMENT SELECTION

Concepts	Modes		
	I	II	III
A	a. spinner	marbles	balls
	b. blocks	blocks	beads
B	a. spinner to spinner	balls to balls	beads to beads
	b. blocks to balls	marbles to blocks	balls to spinner
C	a. spinner	beads	marbles
	b. balls	spinner	blocks
D	a. spinner	beads	blocks
	b. balls	spinner	marbles
E	a. balls	spinner	beads
	b. blocks	marbles	balls
F	a. blocks	spinner	marbles
	b. marbles	blocks	beads

Questions were now written for use in the test.

Probability Tasks

During the development of the probability test, the 36 tasks were numbered in a manner so that the second question in each cell followed

the first question. All six questions for each concept were developed consecutively. The thirty-six protocols are given in Appendix A.

Question Sequence

Each subject had two 15 minute sessions with the examiner. Eighteen questions were administered each session. The two questions in each cell were asked consecutively. Concepts A, C, and E were asked in the first session and concepts B, D, and F in the second. To minimize the teaching effect three controls were used.

1. There was no scoring of responses when the student was present. All conversation between the examiner and the subject was audiotaped. In this way the subject could not interpret whether his response was correct or incorrect by watching the examiner code the responses.

2. During the examination period, the examiner gave no verbal indication of whether the subject had given the expected answer or not.

If the student had received some positive re-enforcement (either 1 or 2 above) for correct responses and no re-enforcement for incorrect responses, a negative climate may have been created. Also if the examiner had given a comment like "good", "fine", or "well done", for a good effort even though the responses were incorrect, the subject may have been led to believe that responses were correct. Reaction of this nature may have caused the subjects to give what they felt were expected responses instead of being free

to answer as they really thought.

3. The subjects were divided into 6 groups of 10 students, 5 boys and 5 girls. The subjects were chosen in the order in which their names appeared on the prepared list. Each of the groups was then assigned to a specific question sequence. This ensured that only 10 students would be examined on a particular concept in each of six question sequences.

Two additional factors arose from sequencing the questions:

a. the subjects were not tested on all six questions in a specific concept consecutively and

b. the sequences of modes were varied from concept to concept. Therefore, the subjects would have difficulty transferring information learned from the questions asked in one cell to the questions asked in a subsequent cell in any given concept.

The six question sequences appear in Table 6. The first examination period for each of the 60 subjects was completed before any second session was started.

Mode Sequence

Because of the hierarchial nature of the three modes it was deemed necessary to vary the sequences in the modes. This sequencing was achieved in the ordering of the questions.

For example, concept A, question sequence 1, has the number 1 for mode I, 8 for mode II, and 6 for mode III. That is, the questions for mode I (1) were asked first, the questions for mode III (6) were

TABLE 6

QUESTION SEQUENCES

Sequence 1				Sequence 2			
Concepts	Modes I	II	III	Concepts	Modes I	II	III
A	1*	8	6	A	8	1	6
B	4	2	9	B	2	4	9
C	7	5	3	C	5	7	3
D	10	17	15	D	17	10	15
E	13	11	18	E	11	13	18
F	16	14	12	F	14	16	12
Sequence 3				**Sequence 4**			
Concepts	Modes I	II	III	Concepts	Modes I	II	III
A	1	6	8	A	8	6	1
B	4	9	2	B	2	9	4
C	7	3	5	C	5	3	7
D	10	15	17	D	17	15	10
E	13	18	11	E	11	18	13
F	16	12	14	F	14	12	16
Sequence 5				**Sequence 6**			
Concepts	Modes I	II	III		Modes I	II	III
A	6	1	8	A	6	8	1
B	9	4	2	B	9	2	4
C	3	7	5	C	3	5	7
D	15	10	17	D	15	17	10
E	18	13	11	E	18	11	13
F	12	16	14	F	12	14	16

* Each number refers to both questions in each cell

asked second and the questions for mode II (8) were asked last. This gave a mode sequence of I - III - II for concept A in question sequence 1. Six mode sequences were determined: I-II-III, I-III-II, II-I-III, II-III-I, III-I-II, III-II-I. Each subject answered questions in each of three mode sequences. In the two sessions, questions on concepts A and B, C and D, E and F, followed the same mode pattern. The six questions of each concept were administered to 30 subjects in

identical order. The mode sequences for each question sequence are shown in table 7.

TABLE 7

MODE SEQUENCES IN RELATIONSHIP TO QUESTION SEQUENCES

Question Sequence 1

Concept	Mode Sequence
A	I - III - II
C	II - I - III
E	III - II - I
B	I - III - II
D	II - I - III
F	III - II - I

Question Sequence 2

Concept	Mode Sequence
A	II - III - I
C	I - II - III
E	III - I - II
B	II - III - I
D	I - II - III
F	III - I - II

Question Sequence 3

Concept	Mode Sequence
A	I - II - III
C	III - I - II
E	II - III - I
B	I - II - III
D	III - I - II
F	II - III - I

Question Sequence 4

Concept	Mode Sequence
A	III - II - I
C	I - III - II
E	II - I - III
B	III - II - I
D	I - III - II
F	II - I - III

Question Sequence 5

Concept	Mode Sequence
A	II - I - III
C	III - II - I
E	I - III - II
B	II - I - III
D	III - II - I
F	I - III - II

Question Sequence 6

Concept	Mode Sequence
A	III - I - II
C	II - III - I
E	I - II - III
B	III - I - II
D	II - III - I
F	I - II - III

The rotation of modes in this manner was important for two reasons.

1. The possibility that a subject could formulate a scheme whereby an assumed correct outcome would be taken from one mode and

directly applied to another mode was reduced.

2. Since each subject was tested in three mode sequences, two concepts each, 30 subjects were tested in each of the six mode sequences.

PILOT STUDY

A pilot study was conducted just prior to the collecting of the data. The test was administered to one grade three student and two grade four students in a manner similar to that previously discussed. However subject responses were not audiotaped.

The purposes of the pilot study were to determine if the protocols were eliciting the kinds of responses that were anticipated, if the verbalization used in the questions was understandable by grade three and four students and if the embodiment selected was suitable for each question.

The pilot study revealed that test items and materials seemed to be consistent with the possible range of abilities of grade three and four students.

ADMINISTRATION OF THE INSTRUMENTS

The instruments were administered during the last week of May and the first two weeks of June, 1976. The Lorge-Thorndike Intelligence Tests and the Canadian Tests of Basic Skills, mathematics portion were administered first to all 60 subjects as a group. The probability test was administered over the next 10 school days. A small room

with a table suitable for testing purposes was provided by the school. The subjects were given as much time as they needed to complete the 18 questions in each session.

DATA COLLECTION

When writing the standardized tests, the subjects recorded their responses on IBM answer sheets. The researcher hand scored these sheets at a later time.

At the beginning of each session of the probability test, each subject was informed that everything that was being said was to be recorded. This discussion was kept at a very informal level. The subjects were asked if they had a tape recorder at home, or if they had ever heard themselves speak on a tape recorder. In some cases, at the beginning of the first session a student spoke into the microphone and then the recording was played back to him. This procedure seemed to make the subject feel at ease. Not one subject objected to the procedure of being recorded. The testing sessions were recorded on 60 minute cassette tapes. The combined time of both sessions for each subject did not exceed 30 minutes. After the data had all been collected the researcher scored each student's responses by listening to the taped conversations. A summary sheet (Figure 2), devised for this purpose, was used to tally individual responses.

A file folder was also kept for each subject. Each individual's answer sheets and summary sheets were filed under questioning sequence and name.

FIGURE 2

SUBJECT'S SUMMARY SHEET

Name: ______________________ Birthdate: ______________________

Question Sequence: __________ Grade: _______ Sex: ___________

Concept	Mode I	Mode II	Mode III	Concept Sum
A	____ ____	____ ____	____ ____	
B	____ ____	____ ____	____ ____	
C	____ ____	____ ____	____ ____	
D	____ ____	____ ____	____ ____	
E	____ ____	____ ____	____ ____	
F	____ ____	____ ____	____ ____	
Mode Sum	________	________	________	

Total Score: __________

Math Score: Concepts _______, Problems _______ (%ile)

IQ: Verbal ____________, Nonverbal ______________

CODING THE DATA

A different coding system was necessary for each of the testing instruments.

a. The Lorge-Thorndike Intelligence Tests.

Using the scoring key provided by the publisher, raw scores were obtained for each subject on the verbal and the nonverbal intelligence tests. The scores were then compared with the table of norms found in the Examiner's Manual to obtain individual intelligence quotients (IQ). These IQ's were recorded on the subjects' summary sheets.

b. The Canadian Tests of Basic Skills.

The raw scores were obtained for both mathematics tests by hand scoring each of the answer sheets. Using the conversion tables found in the Teacher's Manual for the tests, these scores were converted to grade equivalent scores and then to percentile rankings. The percentiles were recorded on the subjects' summary sheets.

c. The Probability Test.

Each correct response received a score of one and each incorrect response received a score of zero. The criteria for scoring correct responses are found in Appendix B. A complete testing session has been transcribed and included in Appendix C. The tallies were then summed to find the concept totals, mode totals, and the test total.

The mode sequence totals were more difficult to tally, therefore separate tables were devised for this purpose. The highest possible score each subject could obtain for any mode sequence was 12.

NULL HYPOTHESES TO BE TESTED

1. Given subsets of scores of the probability test, there is no significant difference between the means for:

a. the high age group and the low age group.

b. the grade three students and the grade four students.

c. the boys and the girls.

2. There is no significant relationship between the scores on the probability test and the scores on the:

a. standardized verbal intelligence test.

b. standardized nonverbal intelligence test.

c. standardized mathematics concepts test.

d. standardized mathematics problem solving test.

3. Given subsets of scores of the probability test, there is no significant relationship between the scores obtained by subjects in:

a. the concrete mode and the pictorial mode.

b. the concrete mode and the symbolic mode.

c. the pictorial mode and the symbolic mode.

4. Given subsets of scores of the probability test, there is no significant difference between the mean of the sequence means and the mean in each of the mode sequences.

a. I - II - III

b. I - III - II

c. II - I - III

d. II - III - I

e. III - I - II

f. III - II - I

5. Given subsets of scores of the probability test, there is no significant relationship between the scores of concepts 1 through 6 when concept scores are considered in pairs.

ANALYSIS DESIGN

Two statistical tests were used to analyze the data. A computer correlation test, DEST 02, was used to determine relationships between the 14 variables in the null hypotheses 2, 3, and 5. The 14 variables for each student were obtained from the subject's summary sheet. They were the subject's:

1. IQ on the verbal intelligence test,
2. IQ on the nonverbal intelligence test,
3. percentile in the mathematics concepts test,
4. percentile in the mathematics problem solving test,
5. score for the concrete mode,
6. score for the pictorial mode,
7. score for the symbolic mode,
8. score for concept A,
9. score for concept B,
10. score for concept C,
11. score for concept D,
12. score for concept E,
13. score for concept F,
14. score for total probability test.

A single run of DEST 02 determined all possible correlations between pairs of the above variables.

A one-way analysis of variance test, ANOV 15, was used to test null hypothesis 1 and 4. Unlike the correlation test individual runs were used to determine if any significant difference was to be found between the means of the scores of the probability tests for subjects'

age, grade, and sex, and for the means of the scores for the six mode sequences.

Chapter 4

FINDINGS AND ANALYSIS OF THE STUDY

Since the hypotheses stated in Chapter 1 and further described in Chapter 3 were analyzed through the use of two basic statistical test styles, the findings and analysis are presented in two sections. The first section contains the findings and analysis that relate to hypotheses 2, 3, and 5. Section two contains a description and analysis of the findings related to hypotheses 1 and 4. The hypotheses were grouped as indicated because of their relationship to statistical correlations and analysis of variance, respectively.

FINDINGS AND ANALYSIS WITH RESPECT TO THE HYPOTHESES

During the testing and data collecting, as described in Chapter 3, certain data were collected for the subjects and subsequently recorded on the individual summary sheets.

The variables and scores tabulated in Table 8 are Age, Grade (GR), Sex, Verbal IQ (V.IQ), Nonverbal IQ (N.IQ), Mathematics Concepts Percentile (MC), Mathematics Problem Solving Percentile (MP), and Probability Test Score (PS).

The data which corresponded to age, grade, and sex were coded for use by the computer as follows:

1. Age. The sample was divided into two age groups: high age group and low age group. The critical age used to divide the sample was 9 years 7.5 months.

TABLE 8

SUMMARY OF SUBJECTS' VARIABLES AND SCORES

Subject	Age	GR.	Sex	V.IQ	N.IQ	MC	MP	PS
1	0	3	0	119	113	56	76	23
2	0	3	0	126	125	14	61	20
3	0	3	0	106	115	34	60	11
4	1	3	0	92	96	43	22	20
5	1	4	0	105	119	55	39	24
6	1	3	1	96	77	17	2	19
7	0	3	1	114	110	23	43	19
8	0	3	1	112	107	14	27	20
9	0	3	1	139	114	82	87	15
10	0	3	1	106	106	76	60	22
11	0	3	0	108	93	12	48	23
12	0	3	0	108	112	38	64	23
13	0	3	0	109	105	24	60	26
14	0	3	0	133	125	95	99	28
15	0	3	0	115	100	6	43	21
16	1	4	1	88	108	4	3	22
17	1	4	1	113	108	55	60	25
18	0	4	1	150	130	99	93	31
19	1	4	1	89	92	5	60	15
20	1	4	1	97	96	36	21	23
21	0	3	0	97	98	38	48	13
22	0	3	0	134	114	43	72	20
23	1	3	0	111	118	70	64	25
24	0	3	0	115	125	76	87	23
25	0	3	0	133	99	67	93	23
26	1	3	1	100	85	14	43	16
27	0	3	1	90	87	43	34	14
28	0	3	1	103	89	12	2	21
29	1	3	1	97	115	56	43	18
30	0	3	1	122	114	76	89	27
31	1	4	0	119	119	87	55	30
32	1	4	0	107	107	23	8	18
33	1	4	0	117	112	77	72	28
34	1	4	0	103	98	50	39	13
35	0	4	0	103	98	47	21	17
36	1	4	1	84	101	8	8	17
37	0	4	1	108	97	13	24	17
38	0	4	1	129	114	55	88	26
39	1	4	1	116	114	55	29	22
40	1	4	1	117	119	69	93	23
41	1	4	0	101	119	40	60	28
42	1	4	0	106	109	8	8	13
43	0	4	0	125	123	69	60	25
44	1	4	0	135	127	99	97	31
45	0	3	0	109	127	43	11	18
46	1	4	1	110	124	47	75	18
47	1	4	1	123	125	51	60	30
48	1	4	1	133	111	69	75	22
49	0	4	1	126	114	40	50	31
50	1	4	1	99	96	5	21	23
51	1	4	0	105	107	40	17	18
52	1	4	0	116	121	87	84	29
53	1	4	0	109	110	2	60	21
54	1	4	0	121	108	36	93	19
55	0	4	0	94	107	36	1	15
56	0	4	1	129	125	77	55	25
57	1	4	1	111	109	33	47	17
58	0	4	1	111	124	16	55	32
59	1	4	1	101	112	55	39	28
60	1	4	1	114	117	91	84	25

The codes for the age groups were:

a. low age group ... 0,

b. high age group ... 1.

2. Grade. Two grade groups naturally arose from the selection of the sample; grade 3 and grade 4. These groups were coded by their corresponding grades:

a. grade three ... 3,

b. grade four ... 4.

3. Sex. In the selection of the sample, two equal groups of students were obtained for the parameter male and female. The codes for the sex groups were:

a. female group ... 0,

b. male group ... 1.

All other variables were entered on the computer cards using the numerical values of the variables.

The Hypotheses Related To Statistical Correlation Coefficients

The Pearson product - moment correlation coefficient was used to obtain a 14 X 14 correlation matrix, Table 9, for the 14 variables identified in Chapter 3. The statistical program, DEST 02, which is a computerized statistical program maintained by the Division of Education Research at the University of Alberta was used to obtain all correlation coefficients. The computer printout also indicated the mean, variance, and standard deviation of the distribution of scores for each of the variables, (Table 10). The

TABLE 9

CORRELATION MATRIX FOR THE FOURTEEN VARIABLES

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1.000	0.594	0.599	0.712	0.434	0.363	0.464	0.133	0.388	0.326	0.298	0.173	0.474	0.491
2		1.000	0.578	0.530	0.451	0.411	0.466	0.227	0.461	0.300	0.224	0.236	0.456	0.516
3			1.000	0.653	0.478	0.354	0.487	0.265	0.393	0.240	0.535	0.030	0.507	0.513
4				1.000	0.351	0.351	0.448	0.188	0.297	0.278	0.318	0.188	0.410	0.450
5					1.000	0.648	0.556	0.424	0.664	0.569	0.596	0.310	0.547	0.837
6						1.000	0.626	0.564	0.657	0.499	0.458	0.659	0.487	0.875
7							1.000	0.579	0.707	0.426	0.581	0.479	0.533	0.866
8								1.000	0.486	0.182	0.282	0.335	0.271	0.613
9									1.000	0.216	0.349	0.389	0.474	0.788
10										1.000	0.234	0.257	0.115	0.573
11											1.000	0.153	0.413	0.635
12												1.000	0.107	0.564
13													1.000	0.608
14														1.000

TABLE 10

MEAN, VARIANCE, AND STANDARD DEVIATION FOR THE FOURTEEN CORRELATION VARIABLES

Variables	Mean	Variance	Standard Deviation
1. Verbal IQ	111.716	188.434	13.727
2. Nonverbal IQ	109.966	138.564	11.771
3. Math Concept %ile	45.016	745.480	27.303
4. Math Problem Solving %ile	51.333	780.285	27.934
5. Concrete Mode	7.450	3.181	1.783
6. Pictorial Mode	7.550	3.581	1.892
7. Symbolic Mode	6.733	4.862	2.205
8. Concept A	5.367	0.632	0.795
9. Concept B	1.917	3.410	1.847
10. Concept C	4.133	2.616	1.617
11. Concept D	4.700	1.910	1.382
12. Concept E	4.783	1.236	1.112
13. Concept F	0.783	1.303	1.142
14. Total Probability Test	21.733	25.596	5.059

critical value of the correlation coefficients needed to reject the null hypotheses at the .05 level of significance is $r > 0.250$.

1. Hypothesis Two.

There is no significant relationship between the scores on the probability test and the scores on the:

a. standardized verbal intelligence test.

b. standardized nonverbal intelligence test.

c. standardized mathematics concepts test.

d. standardized mathematics problem solving test.

After testing this hypothesis using the Pearson product-moment correlation coefficient test, the null hypothesis was rejected at the .05 level of significance.

The correlation coefficients contained in Table 11 indicate that verbal IQ, nonverbal IQ, and the subjects' performances on mathematics concepts and on mathematics problem solving tasks can be used as predictors for the scores obtained on the probability test.

TABLE 11

CORRELATION COEFFICIENTS BETWEEN VERBAL IQ, NONVERBAL IQ, MATHEMATICS CONCEPTS, MATHEMATICS PROBLEM SOLVING, AND THE PROBABILITY TEST SCORES

	V.IQ	N.IQ	MC	MP	PS
V.IQ	1.000	0.594	0.599	0.712	0.491
N.IQ		1.000	0.578	0.530	0.516
MC			1.000	0.650	0.513
MP				1.000	0.450
PS					1.000

The correlations between the variables verbal IQ, nonverbal IQ, mathematics concepts, and mathematics problem solving when considered in pairs, indicate two expected relationships.

a. A correlation exists (r = 0.712) between the subject's verbal IQ and his ability to solve mathematical problems.

Since reading ability is closely related to the ability of a subject to transfer a written mathematical problem to a number sentence, this high correlation could be anticipated.

b. A correlation exists (r = 0.650) between the subjects' performance on the mathematics concepts test and their performance on the mathematics problem solving test.

This expected correlation could reflect the parallel development of mathematical concepts and mathematical problem solving as determined in the curriculum.

2. Hypothesis Three.

Given subsets of scores of the probability test, there is no significant relationship between the scores obtained by subjects in:

a. the concrete mode and the pictorial mode.

b. the concrete mode and the symbolic mode.

c. the pictorial mode and the symbolic mode.

The correlation coefficients that resulted from testing the mode scores in pairs, and the correlation coefficients that resulted when the individual mode scores were compared to the probability test scores are indicated in Table 12. Clearly, the null hypothesis was rejected for each relationship at the .05 level of significance.

Two general statements can be made. There is little difference in the range of the correlation coefficients when the modes are

TABLE 12

CORRELATION COEFFICIENTS BETWEEN THE CONCRETE MODE (CM), THE PICTORIAL MODE (PM), THE SYMBOLIC MODE (SM), AND THE PROBABILITY TEST SCORES

	CM	PM	SM	PS
CM	1.000	0.648	0.556	0.837
PM		1.000	0.626	0.875
SM			1.000	0.866
PS				1.000

considered in pairs (r = 0.556 to r = 0.648). There is also very little difference between the correlation coefficients of the individual mode scores and the probability test scores. The highest correlation coefficient (r = 0.875) is between the performances of subjects in the pictorial mode and their performances on the total probability test.

3. Hypothesis Five.

Given subsets of scores of the probability test, there is no significant relationship between the scores of concepts A through F when the subsets of concept scores are considered in pairs.

The correlation coefficients that resulted from the analysis of the null hypothesis 5 are summarized in Table 13.

The correlation coefficients identified by asterisks indicate all the null hypotheses that were not rejected at the .05 level of significance. All other null hypotheses were rejected.

The concepts were ranked according to the total number of correct responses given by the subjects (Table 14). Since six

TABLE 13

CORRELATION COEFFICIENTS BETWEEN PAIRS OF CONCEPTS AND BETWEEN INDIVIDUAL CONCEPTS AND THE PROBABILITY TEST

	A	B	C	D	E	F	PS
A	1.000	0.486	0.182*	0.282	0.335	0.271	0.613
B		1.000	0.216*	0.349	0.389	0.474	0.788
C			1.000	0.234*	0.257	0.115*	0.573
D				1.000	0.153*	0.413	0.635
E					1.000	0.107*	0.564
F						1.000	0.608
PS							1.000

TABLE 14

INDIVIDUAL CONCEPTS RANKED BY PERFORMANCE

Concepts	Sum Of Scores	Percentage Correct
A	322	89.4
E	287	79.7
D	285	79.2
C	254	70.6
B	112	31.1
F	50	13.9

questions were asked for each concept per subject, each concept had a possible sum of scores of 360.

The concepts: (A) sample space - single outcome event, (E) impossible vs. certain event, (D) more likely vs. less likely event,

and (C) equally likely event, were performed at a level above 70%. This would indicate that students in grade 3 and above have an intuitive understanding of these concepts of probability. The performance of the subjects for the two concepts: (B) sample space-ordered pair outcome event and (F) probability of a simple event indicates that students who are at the end of grade 4 have not acquired an intuitive understanding of ordered pairs or ratios. It can be assumed that this is largely the result of these concepts, ordered pairs and ratios, being introduced into the mathematics curriculum at a higher grade level. Although the performance level for concept D was very low, 31.1%, its correlation coefficient to the scores of the probability test was extremely high ($r = 0.788$). This would seem to indicate that if the students were able to correctly identify the ordered pair outcomes for a probability concept, they were also able to perform well on the overall probability test.

The Hypotheses Related To Analysis of Variance

A one-way analysis of variance test, ANOV 15, was used to analyze the data for hypotheses one and four. ANOV 15 is a computerized statistical program maintained by the Division of Educational Research at the University of Alberta. This program is also designed to provide values of: the Chi Square test for homogeneity of variance, the Scheffe multiple comparison of means, and the Newman-Keuls comparison between ordered means. The results of the Scheffe multiple comparison of means and the Newman-Keuls comparison between ordered means were

not used for analysis of the data.

1. Hypothesis One.

Given subsets of scores of the probability test, there is no significant difference between the means for:

a. the high age group and the low age group.

b. the grade three students and the grade four students.

c. the boys and the girls.

The values of P in the analysis shown in Table 15 indicate that

TABLE 15

ANALYSIS OF VARIANCE ON THE CRITERIA OF AGE, GRADE, AND SEX

Variables		SS	MS	DF	F	P
Age:	Between groups	2.62	2.62	1	0.10	0.754
	Within groups	1533.11	26.43	58		
Grade:	Between groups	85.61	85.61	1	3.42	0.069
	Within groups	1450.13	25.00	58		
Sex:	Between groups	2.40	2.40	1	0.09	0.764
	Within groups	1533.34	26.44	58		

the null hypotheses as stated for age, grade, and sex are not rejected at the .05 level.

For the purposes of the analysis, certain subsets of the sample were determined. The distribution of subjects by age is illustrated in Figure 3. Two relatively equal distributions were achieved when ages were separated at 9 years 7.5 months. Figure 4 shows the comparative sample distributions for age, grade, and sex.

FIGURE 3

FREQUENCY POLYGON FOR SUBJECTS' AGES

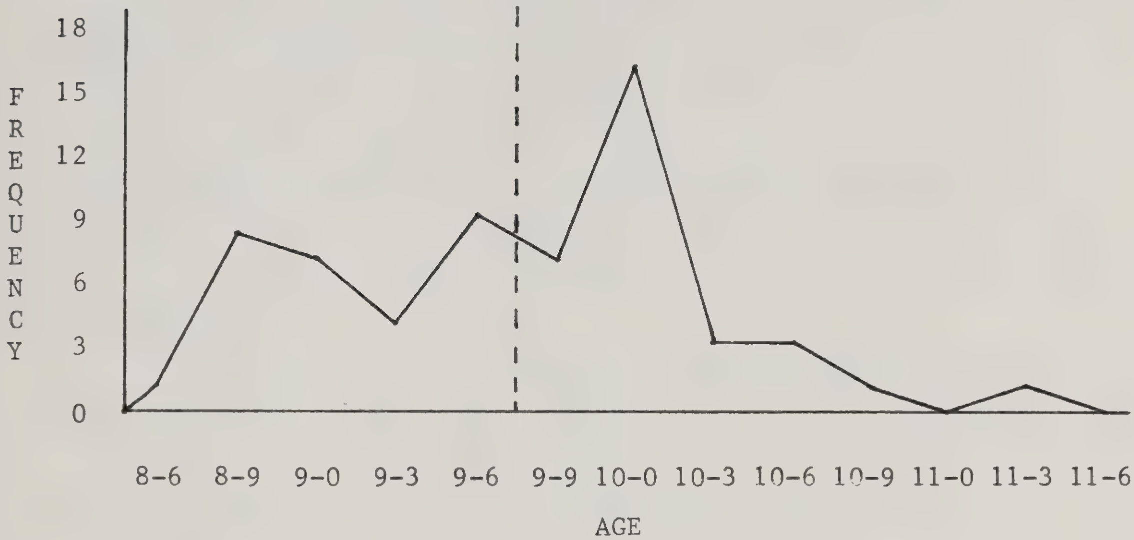
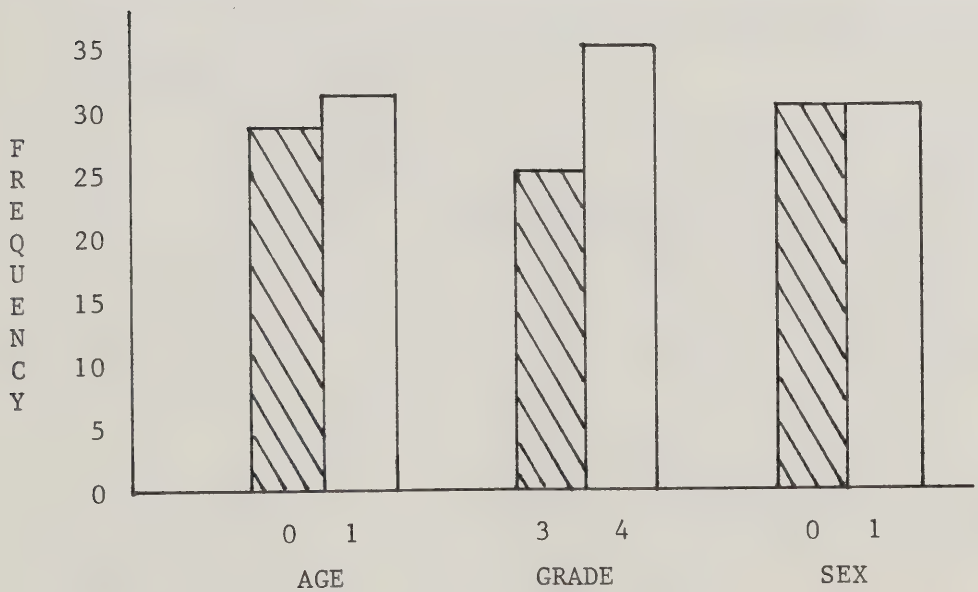


FIGURE 4

FREQUENCY BARGRAPH FOR SUBJECTS AS GROUPED FOR AGE, GRADE, AND SEX



The probability factor (P = 0.069) that resulted as to the effect of grade approaches significance at the .05 level. It is possible to assume that this may be the result of the subjects' years of formal education. Since grade 4 students have one more year in which to experience the formal presentation of mathematics concepts than do the grade 3 students, this effect of grade could be anticipated.

It is noted that the probability that the differences in grade means would occur from random selection is relatively low (P = 0.069), while the probability that the differences in age means is very high (P = 0.754). This appears to be a contradiction. There are two factors, however, which need to be considered.

1. Since the correlation coefficient for age is relatively high, one probable cause is that formal probabiliy concepts are not present in the environmental experiences of a child outside of a formal school experience.

2. The second probable cause is that the sub-sample determined by the variables of age and grade were not similar sub-samples. The frequency distribution for age and grade indicates that age was not a completely reliable indicator of grade. Table 16 shows the cross sampling of age and grade.

The mean score, variance, and standard deviation for each of the six sub-samples determined in this hypothesis and the total probability test are given in Table 17.

TABLE 16

BIVARIATE FREQUENCY DISTRIBUTION FOR AGE AND GRADE

		Age: Low	Age: High	
Grade	3	20	5	25
	4	9	26	35
		29	31	60

TABLE 17

THE MEAN, VARIANCE, AND STANDARD DEVIATION FOR AGE, GRADE, AND SEX

	Mean Score	Variance	Standard Deviation
Low age group	21.517	26.759	5.173
High age group	21.936	26.130	5.112
Grade 3	20.320	18.310	4.279
Grade 4	22.743	29.727	5.452
Female	21.533	29.223	5.406
Male	21.933	23.651	4.863
Total	21.733	25.596	5.059

2. Hypothesis One Prime.

Given subsets of scores of the four concepts: sample space-single outcome event, impossible vs. certain event, more likely vs. less likely event, and equally likely event (concepts A, C, D, E,) there is no significant difference between the means for the grade

three students and the grade four students. The probability factors as determined by the analysis are summarized in Table 18. As indicated in the table, the null hypothesis is not rejected at the .05 level.

TABLE 18

ANALYSIS OF VARIANCE ON THE CRITERIA OF GRADE FOR CONCEPTS A, C, D, & E

	SS	MS	DF	F	P
Between groups	0.791	0.79	1	1.19	0.280
Within groups	0.386	0.67	58		

This hypothesis was formulated because the effect of grade, as determined in hypothesis one, approached significance. In considering this factor, it seemed plausible to determine the effect of grade on the performance of students for the four concepts; A, C, D, E, which were performed at an accuracy level above 70% (Table 14).

The data recorded in Table 18 strengthens the previous discussion which suggested that the student's intuitive understanding of some probability concepts does not relate to the number of years of formal education each student experiences. It further suggests that when the concepts (B, F), which were poorly performed, were included in hypothesis one, the effect of grade became more apparent. It appears that when the subjects were faced with the more difficult probability questions, the grade four subjects had better performance.

3. Hypothesis Four.

Given subsets of scores of the probability test, there is no

significant difference between the overall mode sequence mean and the mean in each of the mode sequences:

a. Concrete mode - pictorial mode - symbolic mode.

b. Concrete mode - symbolic mode - pictorial mode.

c. Pictorial mode - concrete mode - symbolic mode.

d. Pictorial mode - symbolic mode - concrete mode.

e. Symbolic mode - concrete mode - pictorial mode.

f. Symbolic mode - pictorial mode - concrete mode.

The above null hypothesis, as supported by data in Table 19 was not rejected.

TABLE 19

ANALYSIS OF VARIANCE ON THE CRITERIA OF SEQUENCE OF MODES

	SS	MS	DF	F	P
Between groups	7.840	1.57	5	0.24	0.945
Within groups	1139.406	6.55	174		

The high probability obtained in the analysis of the data for hypothesis four may be the result of the limits devised to control the teaching effect in administering the probability test (Chapter 3).

The mean score, variance, and standard deviation for each of the six mode sequences and the overall mode sequence are given in Table 20. The means range from a minimum of 7.033 to a maximum of 7.667.

For the variables of age, grade, sex, or mode sequence, probabilities (Table 21) that have been obtained by the Chi Square

Homogeneity of Variance Test give no evidence that the assumption of homogeneity of variance is violated.

TABLE 20

THE MEAN, VARIANCE, AND STANDARD DEVIATION FOR THE SEQUENCES OF MODES

	Mean Score	Variance	Standard Deviation
I - II - III	7.033	5.670	2.371
I - III - II	7.133	6.051	2.460
II - I - III	7.133	5.706	2.389
II - III - I	7.667	6.989	2.644
III - I - II	7.333	5.402	2.324
III - II - I	7.167	9.523	3.086
Total	7.244	6.374	2.525

TABLE 21

CHI SQUARE HOMOGENEITY OF VARIANCE TEST FOR AGE, GRADE, SEX, AND SEQUENCE

	Chi Square	Probability
Age	0.004	0.949
Grade	1.564	0.211
Six	0.318	0.573
Sequence	3.506	0.622

Chapter 5

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND SUGGESTIONS FOR FURTHER RESEARCH

SUMMARY

The purpose of this research was to identify some basic concepts in probability mathematics and to determine if students of grades three and four had any intuitive understanding of these concepts. It was anticipated that the concepts identified could be sequenced as to their individual suitability for inclusion into the mathematics curriculum for the middle elementary grades.

Twenty-five grade three students and 35 grade four students were selected so as to ensure 30 male and 30 female subjects. Group tests were administered to the sample to obtain subjects' verbal and non-verbal intelligience quotients and their mathematics concepts and mathematics problem solving percentiles. The probability test was then administered to each subject in two interviews with the researcher. Each interview was audiotaped and student responses were scored from the tapes at a later time. Six concepts were identified and tested. Two questions per concept were developed for each of the concrete, pictorial and symbolic modes. In total, the probability test consisted of 36 questions.

The six concepts identified were: A. sample space-single outcome event, B. sample space-ordered pair event, C. equally likely

event, D. more likely vs. less likely event, E. impossible vs. certain event and F. probability of a simple event. Concepts A, C, D, and E were performed above 70% while concepts B and F were performed at 31% and 14% respectively.

On the basis of all data obtained, the effect of age, grade, sex, and sequence of modes on subjects' performance of probability concepts was to be determined. The predictor effect of individual subject's IQ, mathematical abilities and their performance on subtests of the total probability test was also to be identified in relation to their performance on the overall probability test.

CONCLUSIONS

Conclusions With Respect to the Hypotheses

Hypothesis one: According to the results of analyzing the data related to hypothesis one, there is no significant effect of age, grade, and sex on subjects' performance on the probability test. The initial analysis seemed to indicate that the effect of grade ($P = 0.069$) may need to be tested further. Therefore an additional analysis was made on the effect of grade and the subjects' performance of the four concepts that were well understood. The result of this analysis confirmed that the effect of grade ($P = 0.280$) was not significant. It can be concluded that age, grade, and sex had no effect on the subjects' performance on the probability test.

Hypothesis two: The evidence that resulted from the analysis of the data related to this hypothesis indicates that the subjects'

individual verbal IQ, nonverbal IQ, ability to understand mathematics concepts, and ability to solve mathematical problems may serve as predictors for performance on concepts of probability. The correlation coefficients that resulted when the relationship between each of the four variables was tested with the scores of the probability test had a mean of 0.493 with a standard deviation of only 0.030.

Hypothesis three: Contrary to the conclusions in much of the literature, the effect of mode embodiment was not found to be significant in this study. It is recognized however, that two questions for each concept per mode may be a limitation of the study. This limitation may have contributed to this unexpected conclusion. It can be concluded that no one mode embodiment was advantageous to the subjects.

Hypothesis four: The sequence of the modes, as indicated in this hypothesis, was introduced as a control on the teaching effect that could result in testing a number of questions for a particular concept. It was found that the individual mean scores for the six mode sequences did not differ significantly from the overall mode sequence mean. This suggests that there was no measurable teaching effect as a result of the mode sequence used in the questioning procedure of the testing instrument. It can be concluded that the mode sequence used in testing any specific concept was not advantageous to the subjects tested.

Hypothesis five: On the basis of the results from this hypothesis, the following conclusions were determined.

There was found to be no significant relationship at the .05 level of significance between the subjects' scores for the following pairs of concepts (See Table 13).

a. Sample space-single outcome event and equally likely event.

b. Sample space-ordered pair outcome event and equally likely event.

c. More likely vs. less likely event and equally likely event.

d. Probability of a simple event and equally likely event.

e. More likely vs. less likely event and impossible vs. certain event.

f. Impossible vs. certain event and probability of a simple event.

There was a significant relationship determined for each of the nine other pairs of concepts tested.

The performance of the subjects for four of the six concepts, when scored as individual concepts, was 70% or greater. These four concepts were: sample space-single outcome event, equally likely event, more likely vs. less likely event, and impossible vs. certain event. It can be concluded that students, by the time they near completion of grade three, have an intuitive understanding of these four concepts of probability. The performance of the subjects for the other two concepts, when scored as individual concepts was 31% and 14% respectively. These two concepts were sample space-ordered pair outcome event and probability of a simple event. It can be further concluded that an intuitive understanding of these two concepts is not present in students, even when they near completion of their fourth year of formal education.

IMPLICATIONS

Several implications can be drawn from this study for mathematics education. The first relates to the major purpose of the study. Students can understand and in fact have an intuitive understanding of at least four basic concepts of probability by the time they complete grade three. Teachers should plan activities for the development of these concepts during the middle elementary grades. The vocabulary needed is not difficult and in most cases it is being used by the students already.

Students often are led to believe there is only one correct answer for any given question. Certainly this cannot be true in the real world. With the inclusion of probability in the elementary mathematics curriculum, students could be taught the importance of alternative solutions.

The results of hypothesis three imply that any one mode is as successful as another when it comes to presenting probability problems. However, since students react best to a variety of materials and settings, it can be implied that concrete, pictorial, and symbolic materials can be used with equal efficiency or be used at the same time to provide instructional variety.

Since no significant relationship was found between the subjects' scores of all other concepts when tested in pairs, it can be implied that students develop the concept of "equal" earlier in life than they do the other probability concepts. Since "equal" can mean "of the same quantity, size, value" (Webster), students have probably

been exposed to this concept, even in their preschool years. All of the questions asked for this concept in the probability test instrument, centered on the rational fraction concept, one-half. Kieren and Nelson (1977) and Noelting (1978), found that the rational concept of one-half is formulated by children during the first stage in the development of rational number thinking. It is then possible to infer that the concept of equally likely could be introduced, in a formal situation, to students in grades one or two. Squire (1978) determined that students in grade one do intuitively understand the probability concept of equally likely.

The introduction of probability to the elementary curriculum could provide an affective motivation for the students. Although no specific data were recorded on the affective behavior of the subjects as they were being interviewed, some impressions remained with the researcher. The subjects were enthused about the idea of taking the probability test. Their enthusiasm may have been the result of being able to handle new materials, and of being audiorecorded, however such techniques are available and relatively common in most classrooms. Several students were asked if they would be willing to stay and take the test during a recess or noon hour period. Each seemed fascinated by being able to "stay in" and take the test.

This apparent enthusiasm expressed by the subjects toward probability could be utilized by instructors of elementary mathematics. The inclusion of probability in the elementary mathematics curriculum beginning at the middle elementary grades could provide the desired affective change in attitude about mathematics needed by

many elementary students. It would seem that this factor alone would serve as sufficient reason to teach probability.

SUGGESTIONS FOR FURTHER RESEARCH

The main purpose of this study was to survey the level of intuitive understanding of probability concepts of students in the middle elementary grades. No attempt was made to test the feasibility of teaching probability concepts to grades three and four students or to students of any other elementary grade. The main recommendation for further research is that the study be replicated with improvements. The study could be improved by the use of a larger sample, selected from a variety of schools.

A related recommendation with respect to curriculum development could emerge from such replication. An instructional unit or units could be developed using the concepts sample space-single outcome, equally likely, more likely vs. less likely, and impossible vs. certain. Squire (1978) also supports this recommendation. He suggested that Shepler's (1969) proposed sequence for developing research-based curriculum materials using behavioral objectives and task analysis could be used by those who would design such units.

The development of rational number (fraction) concepts has long been connected to a predetermined number of parts of a whole. This whole is most often a pie, geometric figure, or a unit length of a piece of paper. Researchers could use a variety of fractional proportions possible in the probability concepts and develop these into units for teaching of probability but also for the teaching of

fraction concepts. A most interesting study would result through the development of such a unit which could be used in an experimental-control type of study.

When studies of this nature are completed with implications that additional material and concepts should be included in a specified curriculum, the question of available class time becomes a critical issue. With hand held calculators becoming readily available, and their use recommended in elementary mathematics classrooms (NCTM, 1978; NEA, 1977), a study could be designed to determine if the use of calculators would free sufficient time within the present curriculum framework to include the instruction of probability concepts.

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APPENDIX A

QUESTION PROTOCOLS FOR PROBABILITY TEST

1. Concept: Sample space-single outcome.

   Mode: Symbolic.

   Materials: Nil.

   Protocol: There are four yellow tennis balls and two white tennis balls in a bag. If you were to reach into the bag and pull out one tennis ball, what possible colors could it be?

2. Concept: Sample space-single outcome.

   Mode: Symbolic.

   Materials: Nil.

   Protocol: There are five yellow beads, three black beads and two green beads in a jar. You can use the beads to make a necklace for your sister's doll. You can use as many beads as you like and as many colors as you like. What are some ways you can make the necklace?

3. Concept: Sample space-single outcome.

   Mode: Pictorial.

   Materials: An 8" X 10" color picture of three red marbles and two black marbles.

   See Appendix D.

   Protocol: Here is a picture of several different colored marbles. If you were asked to make different groupings of these marbles, what are some possible groupings you could make?

4. Concept: Sample space-single outcome.

   Mode: Pictorial.

Materials: An 8" X 10" color picture of a plastic spinner that has two-fourths green and two-fourths orange.

See Appendix D.

Protocol: Here is a picture of a spinner. If you were to spin the spinner, what possible colors could it stop at?

5. Concept: Sample space-single outcome.

Mode: Concrete.

Materials: One large plastic spinner which is one-half orange and one-half green.

Protocol: Please look at this spinner. You can try it if you like. If you spun the arrow, what possible colors could it stop at?

6. Concept: Sample space-single outcome.

Mode: Concrete.

Materials: A painted block with two sides blue, two sides yellow, and two sides orange.

Protocols: Here is a small painted wooden block. Please look at how it is painted. If you were to roll the block, what possible colors could stop on top?

7. Concept: Sample space-ordered pair outcome.

Mode: Symbolic.

Materials: Nil.

Protocol: Suppose there is in a container on that table, two yellow tennis balls and one white tennis ball. Beside it is a

spinner that is one-half orange and one-half green. If you were to first choose a ball, record the color of the ball, then spin the spinner and record the color the arrow stopped at, what possible combinations of colors could you expect? (The number and color of objects can be repeated)

8. Concept: Sample space-ordered pair outcome.

   Mode: Symbolic.

   Materials: Nil.

   Protocol: Pretend you have two yellow beads and two blue beads in your left pocket, and in your right pocket you have two black beads and one white bead. If you first took a bead from your left pocket and then one from your right pocket, what possible color combinations of beads could you get? (The number and color of beads in each pocket can be repeated)

9. Concept: Sample space-ordered pair outcome.

   Mode: Pictorial.

   Materials: An 8" X 10" color picture of four beads, three red and one white and beside them a blue block.

   See Appendix D.

   Protocol: Here is a picture of four marbles and a painted block. If you were to first select a marble to get one color and then roll the block to get the second color, what possible color combinations could you expect to get?

10. Concept: Sample space-ordered pair outcome.

Mode: Pictorial.

Materials: An 8" X 10" color picture of a spinner, one-third red and two-thirds yellow, and five tennis balls, three white and two yellow.

See Appendix D.

Protocol: If you were to use this material, what possible color combinations could you get by first spinning the spinner and then choosing a tennis ball?

11. Concept: Sample space-ordered pair outcome.

Mode: Concrete.

Materials: Two spinners: the first one is two-quarters green and two-quarters orange, the second one is one-third red and two-thirds yellow.

Protocol: Here are two spinners. You are to spin this one first (point to the green and orange spinner) to get one color, then spin this one second (point to the red and yellow spinner) to get the other color. What possible color combinations could you get by spinning both spinners?

12. Concept: Sample space-ordered pair outcome.

Mode: Concrete.

Materials: A painted wooden block with two sides green, two sides red, and two sides white. Three green beads and two yellow beads.

Protocol: You are again to think of the possible combinations of colors you can obtain by first rolling this block and then

selecting a bead. What could the possible combinations be?

13. Concept: Equally likely.

Mode: Symbolic.

Materials: Nil.

Protocol: Pretend you have ten marbles in a bag. There are only two colors of marbles in the bag. You are to reach into the bag and bring out one marble. If you had an equal chance of getting either color of marble, how many marbles of each color would there be in the bag?

14. Concept: Equally likely.

Mode: Symbolic.

Materials: Nil.

Protocol: (Before asking this question, ensure the subject knows that a block has six sides) You are given a block to paint. If you wanted each color to have an equal chance of stopping on top, when the block is rolled, how many sides would you paint each color?

15. Concept: Equally likely.

Mode: Pictorial.

Materials: An 8" X 10" color picture of six beads, three green and three brown.

See Appendix D.

Protocol: You are to put these beads into a bag and mix them by shaking the bag. You now reach into the bag, get a bead and

record the color. This procedure is repeated many times. What color would be recorded more or would they be recorded about the same number of times?

16. Concept: Equally likely.

Mode: Pictorial.

Materials: An 8" X 10" color picture of five marbles, three red and two black.

See Appendix D.

Protocol: You are to select some of these marbles to put into a bag. You must select them so that if someone would reach into the bag to get a marble, they would have an equal chance of getting either color. How would you select the marbles?

17. Concept: Equally likely.

Mode: Concrete.

Materials: A large plastic spinner, two-fourths green and two-fourths orange.

Protocol: If you spun this spinner many times, how do you think the number of times the arrow stopped on green compared with the number of times it stopped on orange? Would one be more than the other or would they be the same?

18. Concept: Equally likely.

Mode: Concrete.

Materials: Four yellow and three white tennis balls in a large plastic container. Another empty plastic container.

Protocol: You are to select some tennis balls to put into the empty container. You must select them so that if someone were to reach into the container to get a tennis ball, he/she would have an equal chance of getting a yellow ball or a white ball. How would you select the tennis balls?

19. Concept: More likely vs. less likely.

Mode: Symbolic.

Materials: Nil.

Protocol: (Before asking this question ensure the subject knows that a block has six sides) You are given a block to paint. You have only two colors of paint. If you wanted one color to have a more likely chance of stopping on top when the block is rolled, how would you paint the block?

20. Concept: More likely vs. less likely.

Mode: Symbolic.

Materials: Nil.

Protocol: Suppose you have eight marbles in a bag. The marbles are of two colors. When you pick a marble from the bag you know one color is less likely to be picked. How many marbles are there of the less likely color?

21. Concept: More likely vs. less likely.

Mode: Pictorial.

Materials: An 8" X 10" color picture of five brown beads and two yellow beads.

See Appendix D.

Protocol: These beads are put into a bag and they are mixed by shaking the bag. A bead is picked from the bag and its color recorded. If this was repeated many times, how would the number of times a brown bead is picked compare with the number of times a yellow bead is picked? Would it be more, less, or the same?

22. Concept: More likely vs. less likely.

Mode: Pictorial.

Materials: An 8" X 10" color picture of a spinner with one-third red and two-thirds yellow.

See Appendix D.

Protocol: If this spinner was spun many times, what color would the arrow more likely stop on?

23. Concept: More likely vs. less likely.

Mode: Concrete.

Materials: A plastic spinner, one-fourth yellow and three-fourths red.

Protocol: If you spun the spinner many times, how would the number of times the arrow stopped on yellow compare with the number of stops on red?

24. Concept: More likely vs. less likely.

Mode: Concrete.

Materials: Four yellow and four white tennis balls in a large plastic container and another large empty plastic container.

Protocol: You are to select some tennis balls to put into the empty container so that if someone came and took a ball out of the container one color would have a more likely chance of being drawn. How would you select the tennis balls?

25. Concept: Impossible vs. certain.

Mode: Symbolic.

Materials: Nil.

Protocol: You have a bag with six yellow balls in it. If you reached in and took a tennis ball out, what color would you expect it to be? What color would the last ball be? Could you expect to get a white ball?

26. Concept: Impossible vs. certain.

Mode: Symbolic.

Materials: Nil.

Protocol: Suppose there are many beads on the table. There are different colored beads with many beads of each color. You are to select four beads to put into a bag, so that when you reach in and take a bead, you are always certain of what color bead you will take from the bag. How could you select the beads?

27. Concept: Impossible vs. certain.

Mode: Pictorial.

Materials: An 8" X 10" color picture of six yellow and six white tennis balls.

See Appendix D.

Protocol: You are to choose some of the tennis balls to play with. So that you are certain to always play with the same color ball, how would you select a set of four balls to place in the bag?

28. Concept: Impossible vs. certain.

Mode: Pictorial.

Materials: An 8" X 10" color picture of seven yellow marbles. See Appendix D.

Protocol: Pretend that these are your marbles. If you lost a marble, could you lose a green one? Could you lose a yellow one?

29. Concept: Impossible vs. certain.

Mode: Concrete.

Materials: Three painted blocks; one has six sides painted orange, one has three sides white and three sides green, and one has one side each of six different colors.

Protocol: You are to choose one of the three blocks to play with so that you are always certain what color will stop on top. Which block would you choose?

30. Concept: Impossible vs. certain.

Mode: Concrete.

Materials: One large plastic container with four yellow tennis balls in it and another large plastic container with two yellow and two white tennis balls.

Protocol: Which container of tennis balls would you choose if you wanted to be certain of what color of tennis ball you would play with?

31. Concept: Probability of a simple event.

Mode: Symbolic.

Materials: Nil.

Protocol: If you had seven marbles in a bag, five were yellow and two were blue, what would be the chance of losing a yellow marble?

32. Concept: Probability of a simple event.

Mode: Symbolic.

Materials: Nil.

Protocol: If you had ten beads in a bag, five are red and five are green, what chance would you have of reaching in and picking a green bead?

33. Concept: Probability of a simple event.

Mode: Pictorial.

Materials: An 8" X 10" color picture of a spinner, one-fourth yellow and three-fourths red.

See Appendix D.

Protocol: When the arrow is spun, what is the chance it will stop on yellow?

34. Concept: Probability of a simple event.

Mode: Pictorial.

Materials: An 8" X 10" color picture of a painted block, each side painted a different color. The sides showing are orange, yellow, and blue.

See Appendix D.

Protocol: If you were to roll this block, what is the chance of yellow stopping on top?

35. Concept: Probability of a simple event.

Mode: Concrete.

Materials: A spinner, one-third red and two-thirds yellow.

Protocol: If you were to spin the spinner, what is the chance of it stopping on yellow?

36. Concept: Probability of a simple event.

Mode: Concrete.

Materials: A selection of large marbles. Ensure there are four or five marbles of several colors. A small plastic tray (top of container).

Protocol:
a. Choose some marbles and put them into the tray so that they will illustrate a one out of two chance.
b. Choose some marbles and put them into the tray so that they will illustrate a one out of four chance.
c. Choose some marbles and put them into the tray so that they will illustrate a one out of three chance.

APPENDIX B

CRITERIA FOR SCORING STUDENT RESPONSES

The following criteria are arranged in question sequence as outlined in Appendix A. Each criterion indicates all possible solutions and a statement summarizing what the student needed to state to obtain a positive score.

1. Yellow, white: both answers.
2. Combinations of the colors yellow, black, and green: two combinations.
3. Combinations as two black - three red, two black - one red - one red, two red - one black, and one red - one black, etc.: two combinations.
4. Orange, green: both answers.
5. Green, orange (orange is often mistaken for yellow or red): both answers.
6. Blue, orange, yellow: any two answers.
7. Yellow and black, yellow and white, blue and black, blue and white: three answers - order must be maintained.
8. Yellow and orange, yellow and green, white and orange, white and green: three answers - order must be maintained.
9. Red and blue, white and blue (subjects were not told that the black was blue and some indicated it to be black and/or white - their answers could be; red and black, white and black, red and white, white and white): two answers - order must be maintained.
10. Yellow and white, yellow and yellow, red and white, red and yellow: three answers - order must be maintained.
11. Orange and red, orange and yellow, green and red, green and yellow: three answers - order must be maintained.

12. Green and red, green and yellow, orange and red, orange and yellow: three answers - order must be maintained.
13. Five marbles of each color, the same number of each color: one answer.
14. Three sides of each color, or equivalent wording: one answer.
15. About the same, equal: one answer.
16. One black and one red, two blacks and two reds, the same number of each color: one answer.
17. The same, equal, or equivalent wording: one answer.
18. One white and one yellow, two white and two yellow, three white and three yellow, the same number of each color: one answer.
19. Four sides one color and two sides one color, five sides one color and one side one color: one answer.
20. Three, two, one: two answers.
21. More brown beads, or equivalent wording: one answer.
22. Orange (often mistaken for yellow): one answer.
23. More red than orange, red, three times as many red: one answer.
24. Two white and one yellow, three white and one yellow, three white and two yellow, four white and one yellow, four white and two yellow, four white and three yellow: one answer.
25. Yellow, yellow, no: all answers in order.
26. All one color, all the same color, or equivalent wording: one answer.

27. Four tennis balls of the same color, or equivalent wording: one answer.

28. No, yes, all one color: first two answers in order.

29. The all orange block, or equivalent wording: one answer.

30. They are all yellow balls, they are the same color, or equivalent wording: one answer.

31. Five out of seven, or equivalent ratio: one answer.

32. Equal chance, same chance, five out of ten, one out of two, or equivalent ratio: one answer.

33. One out of four, or equivalent ratio: one answer.

34. One out of three, one out of six, or equivalent ratio: one answer.

35. Two out of three, twice as often, or equivalent ratio: one answer.

36. a. Select two marbles - one marble of one color and the other marble of another color,

    b. Select three marbles - one marble of one color and two marbles of another color,

    c. Select four marbles - one marble of one color and three marbles of another color: two answers.

APPENDIX C

SAMPLE INTERVIEW AS TRANSCRIBED

Subject - 33; Sequence - 132.

Part 1 of Interview

Question 1.

E: Pretend you have a bag with four white tennis balls and two yellow tennis balls in it. You are to reach into the bag and pull out a ball. After you look at the color, you put the ball back into the bag so that there is always four white balls and two yellow balls in the bag. What color of tennis ball could you pull from the bag?

S: Yellow or white.

Question 2.

E: Pretend you have 10 small beads. There are five yellow beads, three black beads, and two green beads. That's five yellow beads, three black ones, and two green ones. You want to make a necklace for your doll with these beads. You can use only one bead or all ten beads. What are some ways in which you can make the necklace?

S: You could put two greens, two yellows, and two blacks. You could put a yellow one in the middle, then two blacks next, and then two greens on the outside. Then you could put two yellow ones on the outside, one green one next, and a black one in the middle.

E: Are there many ways to make the necklace?

S: Yes.

Question 17.

E: Here is a container with some tennis balls in it. There are four yellow ones and three white ones in it. Here is an empty container. You are to put some balls from the first container into it. Choose them in such a way that after they have been put into the second container, a person has an equal chance of reaching in and getting a yellow ball or a white ball from the second container.

Pause (while S selects the balls).

E: Tell what you did?

S: I put in two yellow ones and two white ones.

E: Does that give an equal chance?

S: Yes.

E: How else could you do it or is there another way?

Pause (while S reselects tennis balls)

E: Now tell what you did?

S: I put three yellow ones on the bottom and one white one and one yellow one on the top.

E: Does that give an equal chance?

S: Yes, because you would choose between the one yellow one and one white one on top.

Question 18.

E: Here is a plastic spinner. You can spin it to see how it works. If you spun the spinner a large number of times, how do you think the number of stops on green would compare with the number of stops on orange? Would they be the same or

would one be more than the other?

S: One would be more than the other.

E: Why?

S: Because it's pretty hard to make them stop the same.

E: After a large number of spins do you still think it would be that way?

S: Yes.

Question 27.

E: Here is a picture of six yellow and six white tennis balls. In order to be sure you will always draw a certain color from a bag, how would you select a set of four balls to put inside the bag?

S: Put in two yellow ones and two white ones.

E: Would you always know what color you would draw?

S: No.

E: How would you do it then?

S: You would put four yellow ones or four white ones into the bag.

E: This way would you always know what color you would get?

S: Yes.

Question 28.

E: Here is a picture of a set of marbles you can use to play a game of marbles. There are seven yellow marbles and they are all the marbles you have to play with. What chance would you have of loosing a green marble?

S: You couldn't because they are all yellow.

E: What would be your chance of loosing a yellow one?

S: Good.

Question 13.

E: Let's pretend we have a bag with ten marbles in it. There are only two colors of marbles in the bag. If when you would reach into the bag to get a marble you had an equal chance to get either color, how many marbles of each color would there be? There are ten marbles and you have an equal chance.

S: Five of each.

E: How do you know that?

S: To have an equal chance, you have to have the same number of each.

Question 14.

E: How many sides do you think a block has?

S: Four.

E: What if we count the top and bottom as sides? How many then?

S: Six.

E: You are to paint a block. You have only two colors to paint the block. You want to paint it in a way that when you use it you know that either color could stop on top about the same number of times. How would you paint the block?

S: You would paint one side one color and the other side the other color. Then paint another side one color and so on.

E: How many sides would you paint each color?

S: Three.

E: Why would you paint three sides each color?

S: Because if you wanted the same chance, you would paint three sides each color.

Question 29.

E: Here are three blocks. Each one is painted different. See how they are painted.

Pause (Subject picks up the blocks and looks at how they are painted different.)

Choose one of the blocks to play with so that you will always know what color will stop on top. Which block would you choose?

S: This one. (Subject picks up a block)

E: Why do you choose that one?

S: Because its all orange.

Question 30.

E: Here are two containers of tennis balls. You are to choose one of the containers of tennis balls to play with. Which container would you choose if you always wanted to be certain to play with the same color of tennis ball?

S: This one.

E: Why?

S: Because it has four tennis balls that are exactly the same color.

Question 3.

E. Here is another picture of several marbles of different colors. Tell how you could make different groupings of these marbles?

S: Put the two black ones together and the three red ones together.

E: Could you do it another way?

S: You could put one black one and one red one together and one black one and two red ones together.

E: Could you do it another way?

S: You could put the two blacks with the three red ones.

E: Could you do it another way yet?

S: Two black ones with one red one and the two red ones together.

Question 4.

E: Here is a picture of a spinner. What color could the spinner stop on?

S: Orange.

E: Could it stop on any other color?

S: Green.

E: Could it stop on any other color?

S: No.

E: Why?

S: Because there is no other color.

Question 25.

E: Let's pretend you have a container with six yellow balls in

it. If you reached into the bag and took out a ball, what color would you expect it to be?

S: Yellow.

E: If you took out all six balls what color would the last ball be?

S: Yellow.

E: Could you pull a ball out of a different color?

S: No.

Question 26.

E: Suppose there are many beads on the table. There are different colors with many beads of each color. You want to choose four beads to put into a bag, so that when you reach in to take a bead, you always know what color you are going to take out. How would you pick the four beads?

S: You would pick them out of the same grouping, the same color.

E: Could you pick them any other way?

S: Not really.

Question 5.

E: Here is a small painted wooden block. Look at it carefully to see how it is painted. If you were to roll this block what color could stop on top?

S: Orange.

E: Could it stop at any other color?

S: Yes. Blue and yellow.

E: Could it stop at any other color?

S: No.

Question 6.

E: Look at this spinner. What color could it stop at?

S: Green.

E: Any other color?

S: Orange.

E: Any more?

S: No.

Question 15.

E: Here is a picture of some beads. There are three brown beads and three green beads. We put these beads into a bag and mix them by shaking the bag. You reach into the bag and take out one bead at a time. Write the color of the bead on a piece of paper and then replace the bead in the bag. If you did this 20 times which color do you think would be recorded more or do you think they would be recorded about the same?

S: About the same.

E: How did you figure that?

S: If you keep taking one out they would come out about the same because they are all mixed up.

Question 16.

E: Here is another picture of the same marbles only arranged differently. You are to put some of the marbles into a bag, so that if you were to reach in and take a marble you would have an equal chance of picking a red marble or a black marble. How would you choose the marbles to put into the bag?

S: Take two red ones and two black ones.

E: Is there any other way?

S: You could take one red one and one black one.

E: Is there another way yet?

S: No.

Part 2 of Interview

Question 7.

E: Pretend you have a container with one white and two yellow tennis balls in it, one white and two yellows. Beside it there is a spinner that is half orange and half green. First you choose a ball and see what color it is, then you spin the spinner and see what color it stops. What color combinations could you expect to get? There were yellow and white balls and orange and green on the spinner.

S: Green and yellow, white and green, white and orange, and yellow and orange.

Question 8.

E: If you had four beads in one pocket, two yellow beads, and

two blue beads, and in the other pocket you have two black ones and one white one. What color combination could you expect to get by choosing first one bead from one pocket and then a second bead from the other pocket?

S: White and yellow, blue and black, or yellow and black.

Question 23.

E: Here is a spinner. If you spun the arrow many times do you think the arrow would stop more on yellow or on red, or would it stop about the same on both colors?

S: More on red.

E: Why?

S: Because three-quarters of the circle is red and only one-quarter is yellow.

Question 24.

E: Here is a collection of tennis balls in a box. You are to put some of the tennis balls into this other container so that there is a more likely chance to choose a white ball.

S: O.K.

E: What have you chosen?

S: Two yellow ones and four white ones.

E: How else could you have chosen them?

S: I could have put just the four white ones in.

E: Could you have done it another way?

S: No.

Question 33.

E: Here is a picture of a spinner. What do you think is the chance of the arrow stopping on orange?

S: About ten to two.

E: Is there another way?

S: No.

Question 34.

E: Here is a picture of a painted block. What chance do you think it has of stopping on yellow when it is rolled?

S: Not much.

E: Can you suggest a number for it?

S: About, 10 to about 3.

Question 19.

E: Suppose you had to paint another block. Do you remember how many sides a block has?

S: Six.

E: You have two colors of paint. You want one color to have a more likely chance than the other color to stop on top when the block is rolled.

S: One color is to have a more likely chance than the other color. One side - one color and five of the other color.

E: Is there any other way?

S: Two of one color and then four of the other color.

Question 20.

E: Suppose you had eight marbles in a bag. You choose a marble and you know one color is less likely to be drawn than the other color. How many marbles could there be of the less likely color?

S: Three.

E: Any other number?

S: One or two.

E: Anything else?

S: I don't think so.

Question 35.

E: Here is a spinner. What do you think are the chances of the spinner stopping on orange?

S: Good.

E: Do you have a number you could give to it?

S: About ten to eight.

Question 36.

E: Here is a container of marbles. I want you to illustrate some things for me with the marbles.

S: What do you mean by that?

E: You are to choose some marbles when I give you a chance I want to draw a certain color. You can set the marbles up over here to make it true. If you had a one out of two chance to get a certain color, how could you show it?

S: Which color?

E: That doesn't matter. Whatever color you want to choose.

Pause (Subject choose marbles).

E: What did you choose?

S: A black and a clear.

E: Can you illustrate a one out of four chance?

S: I still don't understand what you mean.

E: If these marbles were put into a bag, and you were to reach in and take out a marble you would have a one in four chance of getting a certain color.

S: One black one and three clear ones.

Question 9.

E: Here is a picture of four marbles and a block. If you were to first pick a marble to get a color and then roll the block to get a second color, what color combinations could you get? Suppose the block is black and white.

S: Red and black, red and white, or white with a black, or white with a white.

Question 10.

E: If you spin the spinner first and then choose a tennis ball, what might the color combinations be?

S: White with an orange, white with a red, yellow with an orange, or a yellow with a red.

Question 31.

E: Suppose you had seven marbles, five of them were yellow and two of them were blue. What chance would you have of losing a

yellow marble?

S: Do you mean like losing it in a game?

E: It won't matter how. Say just losing it. If you lost one marble, what chance would you have of losing a yellow one?

S: Pretty good chance.

E: About what number do you think it is?

S: (no response)

Question 32.

E: You have ten marbles, five are red and five are green. You put the marbles into a bag. If you were to reach into the bag and pull out a marble, what chance would you have of getting a red marble.

S: About a ten to five chance.

Question 11.

E: Here are two spinners. You are to spin this one first to get one color then spin this one second to get the other color. What color combination can you get by spinning this spinner first and the other one second?

S: Green and red, orange with a yellow, green with a yellow, or an orange with a red.

Question 12.

E: You have this block and this group of beads. You are to roll the block first and then pick a bead. What color combination could you get then?

S: Green with a green, a white with a green, a red with a green, yellow with a green, or a yellow with a white, or a yellow with a red.

Question 21.

E: Here is a picture of some beads. If these beads were mixed in a bag and you reached into the bag and picked out a bead. Write the color down then put the bead back into the bag. Do you think you would pick more yellows, more browns, or would they be about the same?

S: I think you would pull out the brown ones most of all.

E: Why?

S: Because there are five brown ones and only two yellow ones.

Question 22.

E: If you had this spinner and spun it many times, do you think it would stop more on the red, more on the orange or do you think they would be about equal?

S: I think more on the orange.

E: Why?

S: There are two-thirds orange and one-third is red.

E: Thank you.

APPENDIX D

COLOR PICTURES USED IN THE PROBABILITY TEST

The following set of pictures are not exact replicas of those used in the study. The apparatus seen in each picture corresponds with the apparatus photographed for use in the prabability test with one exception. There was one additional yellow tennis ball in the picture used with question 27. These pictures were photographed after the testing was completed.

Picture used for question 3.

Picture used for question 4.

Picture used for question 9.

Picture used for question 10.

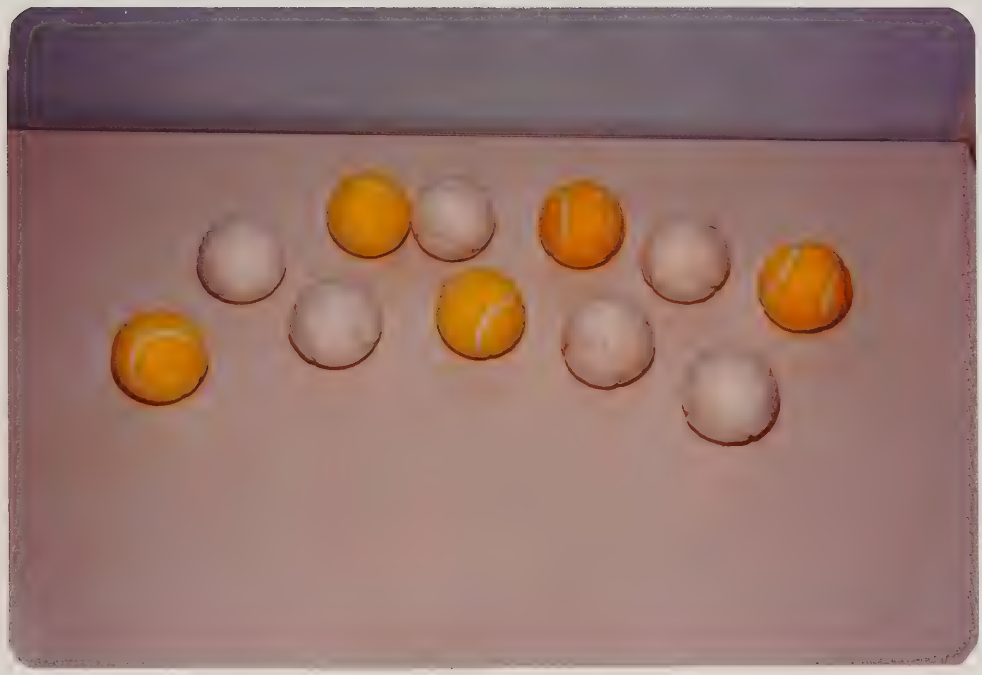
Picture used for question 15.

Picture used for question 16.

Picture used for question 21.

Picture used for question 22.

Picture used for question 27.



Picture used for question 28.

Picture used for question 33.

Picture used for question 34.